Factorization and Quarkonium Production at High p_T

Quarkonium Working Group 2016
PNNL, Jun. 6, 2016
George Sterman, Stony Brook

A walk through evolving formalism

with Z.B. Kang, Y.Q. Ma, G.C. Nayak, J.W. Qiu & H. Zhang (2005 –)

I. Motivation: Color singlets, octets and NRQCD

II. Leading Power Factorization and NRQCD

III. Factorization and NRQCD Beyond Leading Power

IV. Fragmentation Functions and Their Evolution

V. Conclusions

References

[1] Y. Q. Ma, J. W. Qiu, GS and H. Zhang, arXiv:1509.05907 [hep-ph].

[2] Z. B. Kang, Y. Q. Ma, J. W. Qiu and GS, Phys. Rev. D 91, no. 1, 014030 (2015) [arXiv:1411.2456 [hep-ph]].

[3] Y. Q. Ma, J. W. Qiu, GS and H. Zhang, Phys. Rev. Lett. 113, no. 14, 142002 (2014) [arXiv:1407.0383 [hep-ph]].

[4] Z. B. Kang, Y. Q. Ma, J. W. Qiu and GS, Phys. Rev. D 90, no. 3, 034006 (2014) [arXiv:1401.0923 [hep-ph]].

[5] Z. B. Kang, Y. Q. Ma, J. W. Qiu and GS, PoS Hadron 2013, 136 (2013).

[6] Z. B. Kang, J. W. Qiu and GS, Phys. Rev. Lett. 108, 102002 (2012) [arXiv:1109.1520 [hep-ph]].

[7] Z. B. Kang, J. W. Qiu and GS, Nucl. Phys. Proc. Suppl. 214, 39 (2011).

[8] S. Fleming, A. K. Leibovich, T. Mehen and I. Z. Rothstein, Phys. Rev. D 87, 074022 (2013) [arXiv:1301.3822 [hep-ph]].

[9] S. Fleming, A. K. Leibovich, T. Mehen and I. Z. Rothstein, Phys. Rev. D 86, 094012 (2012) [arXiv:1207.2578 [hep-ph]].

[10] G. C. Nayak, J. W. Qiu and G. F. Sterman, Phys. Rev. D 72, 114012 (2005) doi:10.1103/PhysRevD.7

[11] G. C. Nayak, J. W. Qiu and G. F. Sterman, Phys. Lett. B 613, 45 (2005) doi:10.1016/j.physletb.2005.03.031 [hep-ph/0501235].

I. Motivation: Color singlets, octets and NRQCD

- **The context: almost-but-not-quite-complete success of Nonrelativistic QCD (NRQCD) in describing high-p_T heavy quarkonium production at hadron colliders.** (viz. Polarization, associated production)

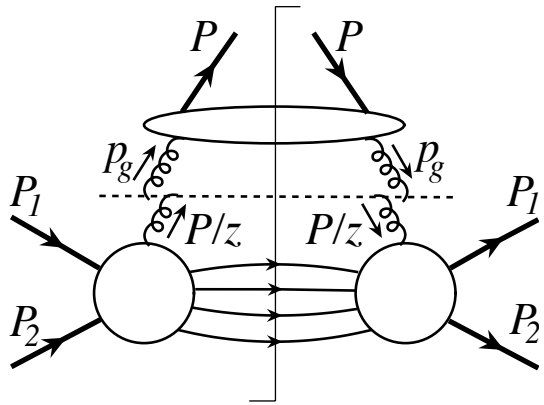
- **The intriguing puzzles of factorization in quarkonium production.**

- **Large NLO, NNLO corrections associated with color configurations previously thought to be small** (Artoisenet, Campbell, Lansberg, Maltoni, Tramontano: 2007,8).

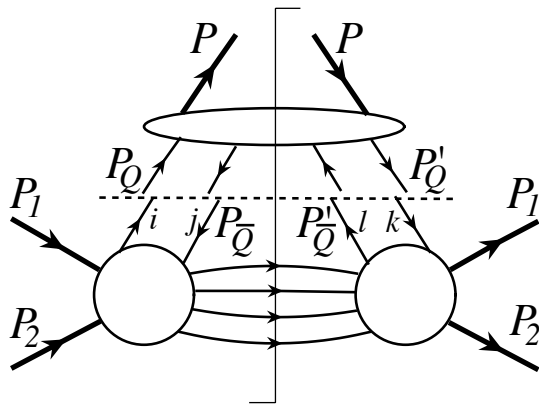
- **Size of color singlet cross sections seems to upset long-held expectations for gluon fragmentation/color octet dominance**

- **Do we need to widen the NRQCD formalism? Can we?**

- **In a nutshell: it's useful to develop a formalism that incorporates both**



- **and**



- **Despite the fact that gluon fragmentation dominates by a power of p_T, eventually.**

Background of NRQCD: historical sketch

- **What makes heavy quarkonium decay and production special?:**
  The annihilation/creation of valence quarks is perturbative.

- **What of hadronization?**
  Color state at annihilation/creation?

- **"Color singlet model (CSM)":**
  pair is neutral right down/up to short distances.

- **CSM limitations:**
  - **IR divergences in "P-wave" decay width**
  - **Rates of high-p_T HQonium production**

The NRQCD solution (Bodwin, Braaten and Lepage 1994) (BBL)

- **Work in HQonium rest frame:**
  "Integrate out" HQ dynamics ($\mathcal{O}(\alpha_s^n(m_Q))$)
  separating nonrelativistic quark & antiquark fields ψ & χ from the full Dirac field.

- **Factorize hadronization and expand in orders of v_{rel} $\Rightarrow$ NRQCD**

- **Color octet** *and* **color singlet $\psi\chi$ $\Rightarrow$ HQonium. Octet appears with $1/p_T^4$ at LO in the coefficient function, while singlet behaves as $1/p_T^8$ at LO.**

- **Symmetries provide many predictions**
  (the hallmark of effective theories)

- **The remaining issue – when is the effective theory "effective"?**
  (the essential question for effective theories)

II. Leading-power Factorization and NRQCD

Collinear factorization and fragmentation

- **Heavy quarkonium production at high p_T**
- **Leading power: factorization into fragmentation function**
  (A classic result, clarified for hadroproduction in Nayak, Qiu, GS, (2005) (NQS))

$$d\sigma_{A+B\to H+X}(p_T) \;=\; d\hat{\sigma}_{A+B\to g+X}(p_T/z\mu) \otimes D_{H/g}(z, m_c, \mu) \;+\; \mathcal{O}(m_c^2/p_T^2)\,,$$

plus contributions of other light partons.

- **With $D_{H/g}$ defined as a VEV:**

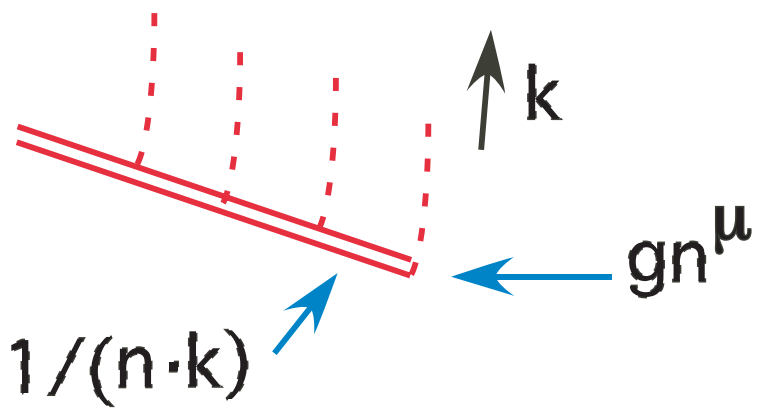
$$D_{H/g}(z, m_c, \mu) \;\propto\; \frac{1}{P^+}\mathrm{Tr}_{color}\int dy^- \mathrm{e}^{-i(P^+/z)y^-}\langle F^{+\lambda}(0)\,[\Phi_n^{(g)}(0)]^\dagger\, a_H^\dagger(P^+)\, a_H(P^+)\,\Phi_n^{(g)}(y^-)\, F_\lambda^+(y^-)\rangle$$

- **With "Wilson lines" $\Phi_n(x^-)$ in direction "n".**

- **The Wilson line [a.k.a. path ordered exp, nonabelian phase, eikonal line] in x^- direction ($n^\mu = \delta_{\mu-}$):**

$$\Phi_n^{(g)}(x^-) = P \exp\left[-ig \int_0^\infty n \cdot A^{(adj)}\left((x^- + \lambda)n\right)\right]$$

- **To the jet, all that's left of the rest of the world is gluon source! Fragmentation analog of "current quark" in DIS.**



- **And what NRQCD says for production:**

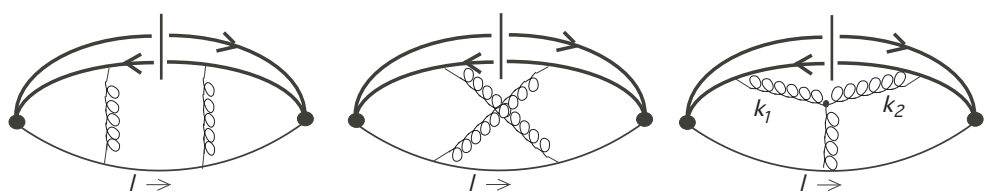
$$d\sigma_{A+B\to H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\to c\bar{c}[n]+X}(p_T)\,\langle \mathcal{O}_n^H\rangle$$

- **With VEVs of the "production" operators (BBL 1994, NQS 2005)**

$$\mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_{nc}\psi(0)\,\Phi^\dagger_{l,cb}(0)\left(a_H^\dagger a_H\right)\,\Phi_{l,ba}(0)\psi^\dagger \mathcal{K}'_{n\ a}\chi(0)$$

Here light-like direction l is arbitrary, at least to NNLO.

- **What needed to be checked, and remains to be generalized: diagrams like these:**



can have

$$\alpha_s^2\,\frac{\vec{v}^{\,2}}{\varepsilon^2}$$

but not (NQS 2005)

$$\alpha_s^2\,\frac{(\vec{v}\cdot\vec{l})^2}{\varepsilon^2}$$

- **This is not obvious at even at NNLO. Is the theory really "effective"?**

- **Nevertheless, what we conclude by assuming the extension of this result, and combining it with the basic LP factorization is**

$$D_{H/g}(z, m_c, \mu) = \sum_n d_{g\to c\bar{c}[n]}(z, \mu, m_c) \langle \mathcal{O}_n^H \rangle$$

with perturbative $d_{g\to c\bar{c}}$.

- **The details of the operator definitions of the matrix elements are not essential to use this conjecture, it is their "universality" that is important.**

- **It was recognized early on that this form implies that at high p_T color and polarization states of the pair reflect those of the gluon – color octet and transverse. Corrections that involve singlet matrix elements are high order and small. Similarly for longitudinal polarizations.**

- By now LP fits to the $\langle \mathcal{O}_n^H \rangle$ for $\mathcal{K} \to {}^3S_1^{[1,8]}$, ${}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$ operators, with exact NLO calculations and NP evolution. NLO is absolutely necessary. Many authors are here!

  ...Butenschoen and Kniehl (2012); Chao, Ma, Shao, K. Wang, Y.J. Zhang (2012); Gong, Wan, J.-X. Wang, H.-F. Zhang (2013); Bodwin, Chung, Kim, Lee (2014); Faccioli, Knunz, Lourenco, Seixas, Wohri (2014); Bodwin, Chao, Chung, Lee, Ma (2015) ...

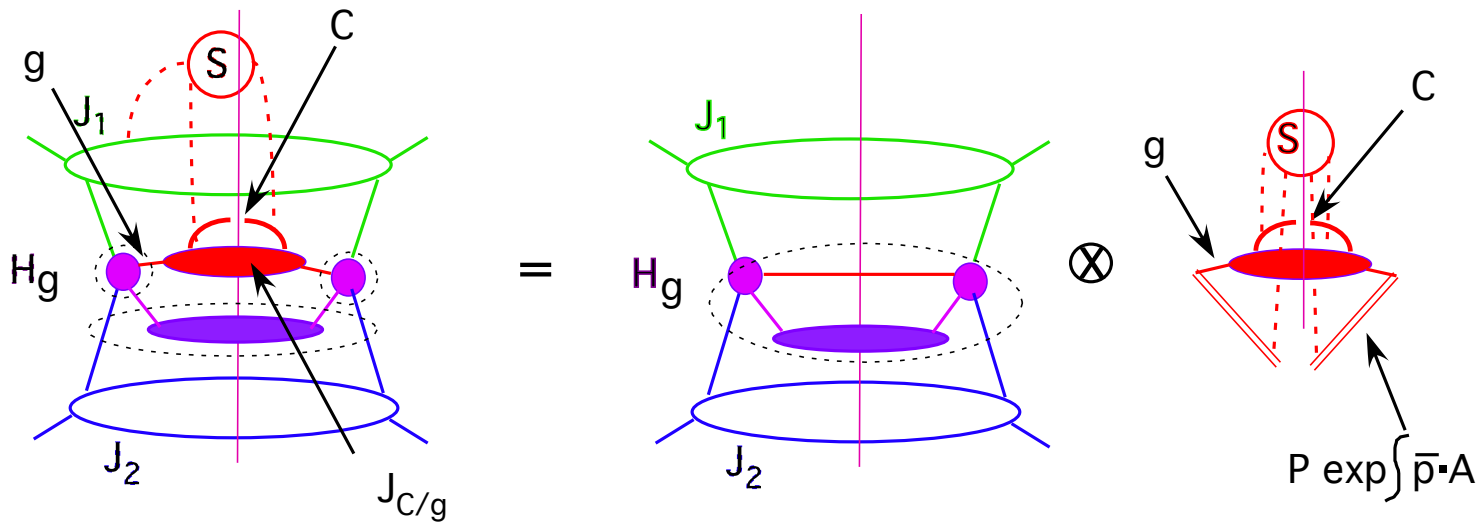
  The issues that remain might be summarized as

- Polarization
- The emergence of color singlet contributions at NLO
- The "global" picture for hadron collisions, deep-inelastc scattering and leptonic annihilation to hadrons

The extension to NLP factorization is an attempt to contribute to an eventual synthesis of all these data.

III. Factorization and NRQCD Beyond Leading Power

- **Where leading power factorization comes from . . .**
- **Systematic identification of "pinch surfaces": lower-dimensional subspaces of momentum space that lead long-distance sensitivity. This is how we know leading power is really leading to all orders in PT.**
- **The general picture in "cut diagram notation":**



$$\sigma_{1+2\to C+X} = \tilde{\sigma}_{1+2\to g+X} \otimes D_{C/g}$$

- **Fragmenting parton sees gluons only in "universal" way. Other effects cancel at LP.**

- **Leading power (LP) is a self-consistent formalism. But it doesn't capture everything. There are also nonleading pinch surfaces, which do not produce leading power (NLP), but would induce new scales dynamically, divided by powers of the large momentum transfer. Can "usually" be neglected, but always?**

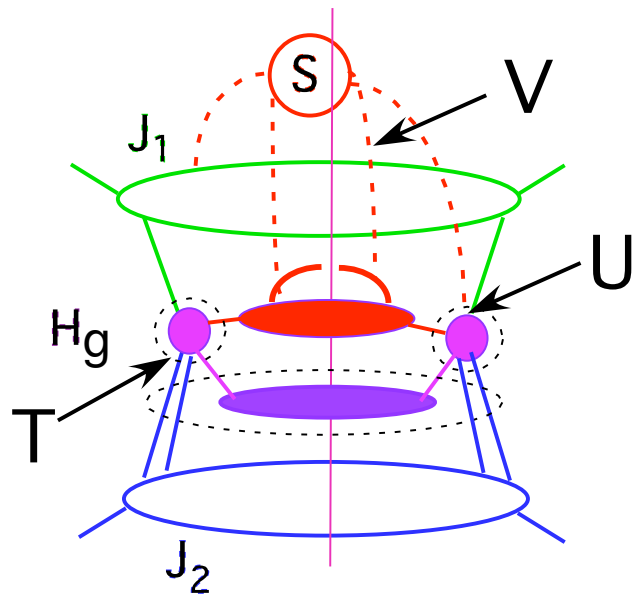
- **May depend on the size of dynamical matrix elements, and role in evolution.**

- **Could this scale be m_Q for heavy quarkonium?**

- **For scattering amplitudes involving external hadrons, a formalism has existed for quite some time (Brodsky & Lepage; Efremov & Radyushkin, circa 1980.) How can we enlarge the formalism for cross sections?**
  (Qiu, GS, 91 for initial state effects. Guo, Wang, 2000)

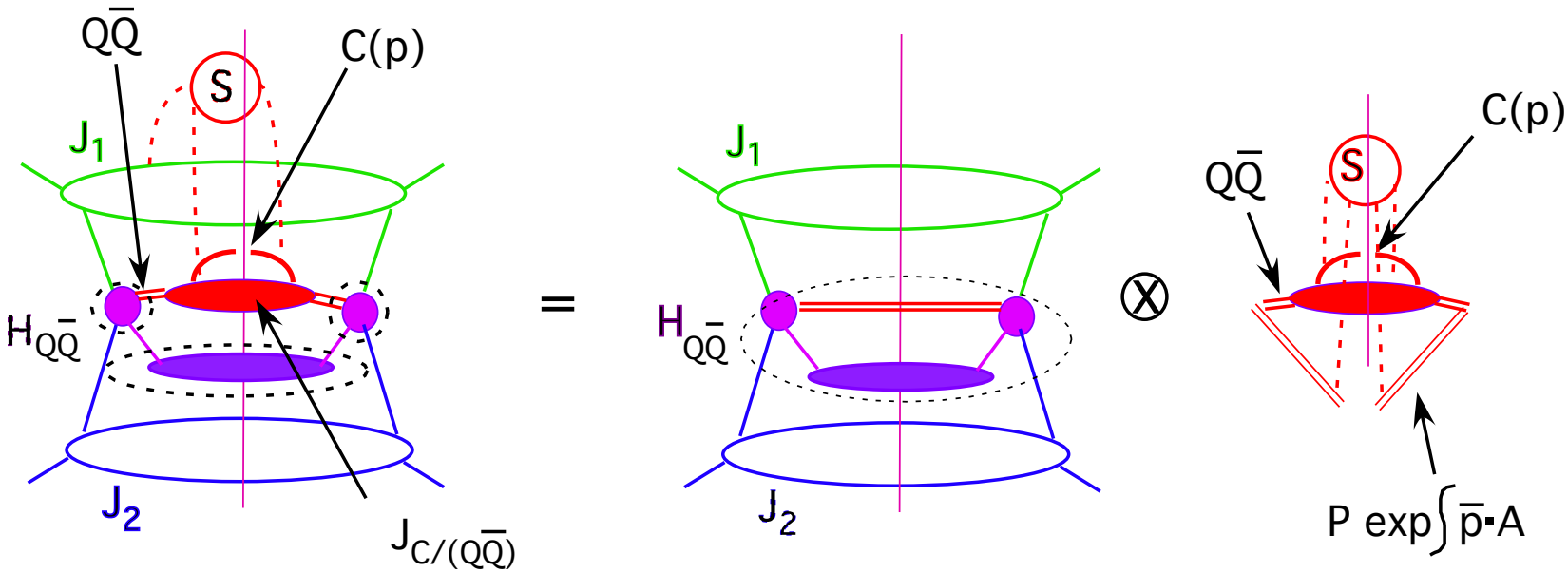
- **Interesting note: related considerations have surfaced recently in NLP threshold resummation (Bonocore, Laenen, Magnea, Melville, Vermaza, White 2015) for cross sections and in soft photon and graviton theorems for ampliudes (Larkoski, Neil, Stewart 2014).**

- **Nonleading powers include many interesting effects that are probably not "big" at hadron colliders . . .**
- **And which certainly don't directly relate to calculation of quarkonia production . . .**



- **T: multiparton initial state, U: soft lines emitted from short distances, V final-state interactions that influence fragmentation (non-factoring, power suppressed)**

- **We study: power correction from pair creation at short distances. Suppressed by dimensional counting.**
- **The corresponding leading regions:**



$$\sigma_{1+2 \to (Q\bar{Q})+X} = \tilde{\sigma}_{1+2 \to (Q\bar{Q}+X)} \otimes D_{C/(Q\bar{Q})}$$

- **Still "factorized" with even smaller corrections. It's additive to leading power.**

- **Extended factorization:**

$$d\sigma_{A+B\to H+X}(P) \approx \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = P/z, \mu_F) \otimes D_{f\to H}(z, m_Q, \mu_F)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to [Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]} = P/z, \{z_i\}, \mu_F)$$
$$\otimes \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z, \{z_i\}, m_Q, \mu_F) + \mathcal{O}\left(\frac{m_H^4}{p_T^4}\right) ,$$

- **With corrections that are also $1/p_T^2$ – but added to leading-power gluon fragmentation**

- **At $1/p_T^4$ expect "non-factoring" corrections linking initial and final states.**

- **A very similar formalism has been developed in Soft Collinear Effective Theory (Fleming, Leibovich, Mehen, Rothstein, 2013).**

IV. Fragmentation Functions and Their Evolution

$$D_{[Q\bar{Q}(a8)]\to HX}(z, m_Q, \mu_F) = \left(\frac{z^2}{4P^+}\right) \sum_X \int \frac{dy^-}{2\pi}\, \mathrm{e}^{-i(P^+/z)y^-}$$
$$\times \frac{4}{N_c^2-1} \sum_{a,b,c=1}^{N_c^2-1} \langle 0|\, \psi(0) \frac{\gamma^+\gamma_5}{2}\, t^a\, \overline{\psi}(0) \left[\Phi_n^{(A)}(0,\infty)\right]^\dagger_{ab} |H(P^+)X\rangle$$
$$\times \langle H(P^+)X| \left[\Phi_n^{(A)}(y^-,\infty)\right]_{bc} \psi(y^-) \frac{\gamma^+\gamma_5}{2}\, t^c\, \overline{\psi}(y^-)\, |0\rangle\,,$$

- **Also have:**

  $D_{[QQ(v\,8)]\to HX}$ **w/out the γ_5s, and**

  $D_{[QQ(a,v\,1)]\to HX}$ **w/out the t^as.**

- **Useful for $\mu_F > m_Q$. Below that, match to NRQCD**
  (Ma, Qiu, Zhang (2013, 14, 15))

- **These distributions are all boost invariant, hence "evolve":**

$$D(z,\mu) \sim \sum_p \int^\mu \frac{dk_T^2}{k_T^p} \times \gamma_p(z) \;\; p = 2,\; 4$$

- **General evolution mixes LP with NLP,**

$$\begin{aligned}
\frac{d}{d\ln\mu_F^2} D_{f\to H}(z, m_Q, \mu_F) &= \sum_i \gamma_{f\to i}(z,\alpha_s) \otimes D_{i\to H}(z, m_Q, \mu_F) \\
&+ \frac{1}{\mu_F^2} \sum_{[Q\bar{Q}(\kappa)]} \gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\alpha_s) \otimes D_{[Q\bar{Q}(\kappa)]\to H}(z, m_Q, \mu_F)\,, \\
\frac{d}{d\ln\mu_F^2} D_{[Q\bar{Q}(c)]\to H}(z, m_Q, \mu_F) &= \sum_{[Q\bar{Q}(\kappa)]} \Gamma_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\alpha_s) \otimes D_{[Q\bar{Q}(\kappa)]\to H}(z, m_Q, \mu_F)
\end{aligned}$$

- **The matrix element definition of $D_{Q\bar{Q}\to H+X}$ makes possible computation of (1) splitting functions, (2) hard scattering functions, independently of NRQCD. Direct QCD and SCET approaches agree in most aspects.**

- **At least as I understand it, however, the nonleading power mixing is not a natural feature in the SCET-based formalism. This is presumably a choice, but one which may affect how gluon splitting at intermediate scales $Q \gg \mu \gg m_Q$ is organized.**

V. Conclusions

- **All elements are available for a new phenomenology with nonleading powers.**

- **First expand in p_T (factor); then in α_s**

- **Play off the size of matrix elements against power suppression.**

- **Contact with NRQCD in models of low-scale fragmentation functions for $\mu_F \sim m_Q$.**
  (Y.Q. Ma, J.W. Qiu, H. Zhang (2013-15))