Stoponium on the lattice

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based on PRD92 (2015) 094505 (arXiv:1509.07080)
and work in progress

Outline

1. Introduction
2. Result
3. Conclusion

stopnonium: stop–anti-stop bound state

• bound states of heavy scalar quark ($m \sim m_t$) under the strong interaction

• active searches at LHC experiment (e.g., ATLAS collab., JHEP1310, 189 (2013), N.Kumar and S.P. Martin, PRD90, 055007 (2014), recently, B. Batell and S. Jung, JHEP1507, 061 (2015))

• S-wave stoponium production/decay and NRQCD

$$d\hat{\sigma}(ab \to \Psi + X) = \sum_n d\hat{\sigma}(ab \to \tilde{t}\bar{\tilde{t}}[n] + X)\langle O^{\Psi}[n]\rangle, \tag{1}$$

where

$$\langle O^{\Psi}[n]\rangle \simeq \langle 0|\chi^{\dagger}\psi|1S\rangle\langle 1S|\psi^{\dagger}\chi|0\rangle = |\langle 0|\chi^{\dagger}\psi|1S\rangle|^2 \tag{2}$$

stopnonium: stop–anti-stop bound state

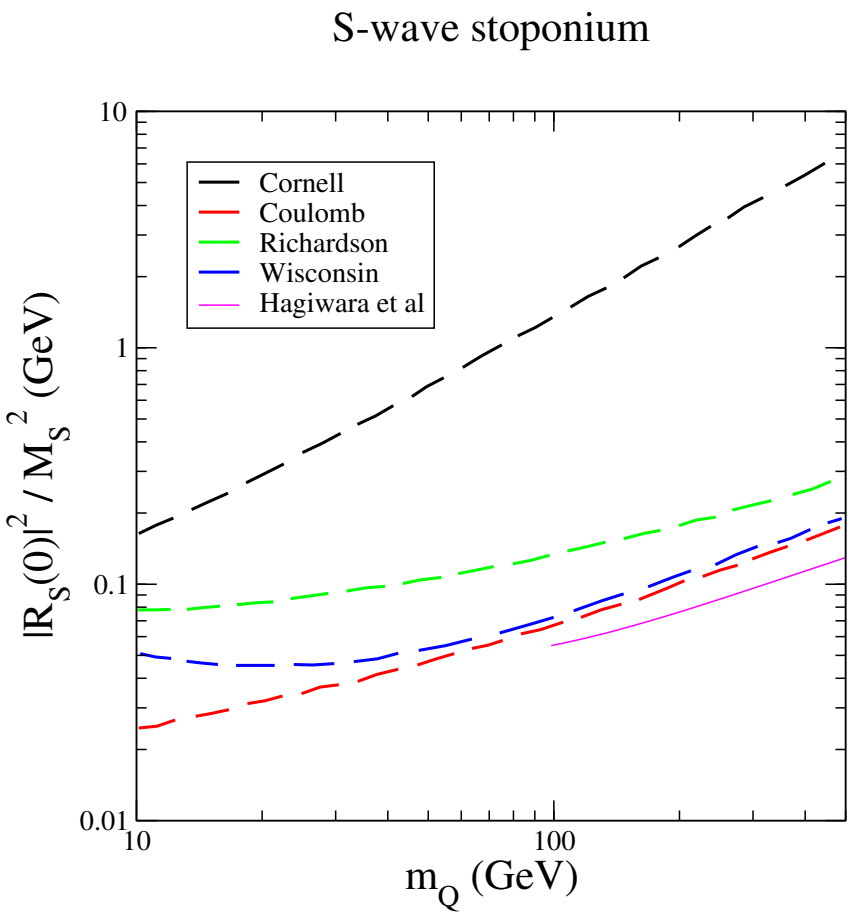
- scale separation: M, Mv, Mv^2
- decay rate = perturbative parts $\times$ non-perturbative matrix elements
- similar to positronium decay
- potential model and wave-function: $|R_{1S}(0)|^2/M_{1S}^3$ and $|R'_P(0)|^2/M_P^5$

$$i\,\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \mathcal{H}\psi \tag{1}$$

with $\mathcal{H} = \frac{\mathbf{p}^2}{M} - V$

stopnonium: stop–anti-stop bound state

- potential model results: Barger et al, PRD35, 3366 (1987)



- improved potential model (?): Hagiwara et al, NPB344 (1990) 1

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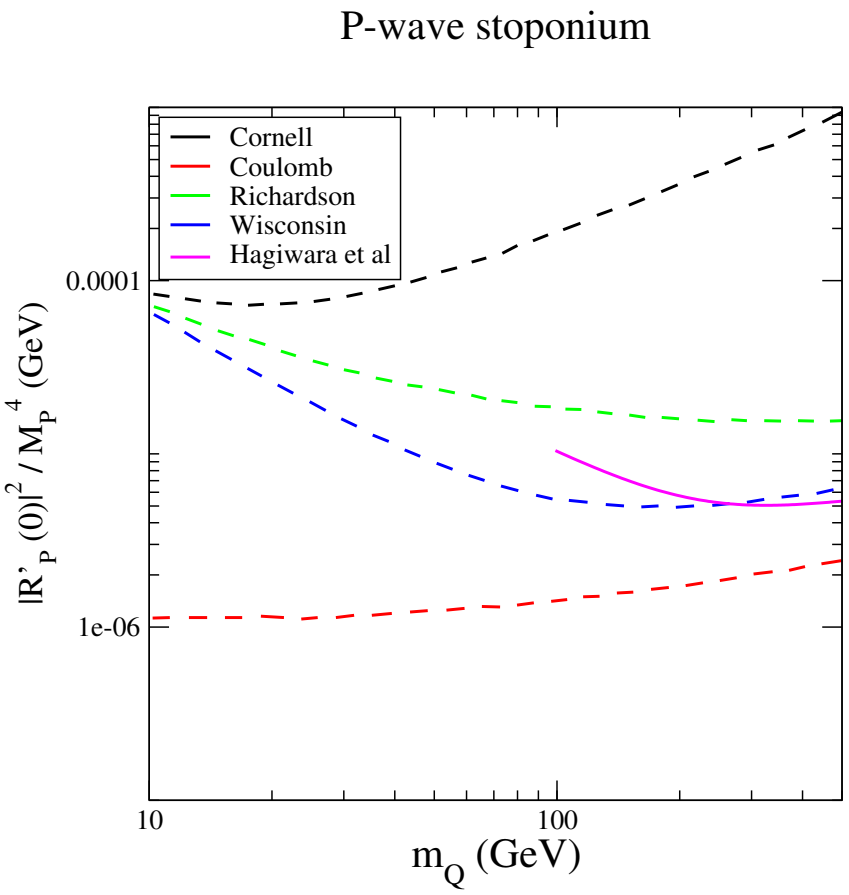

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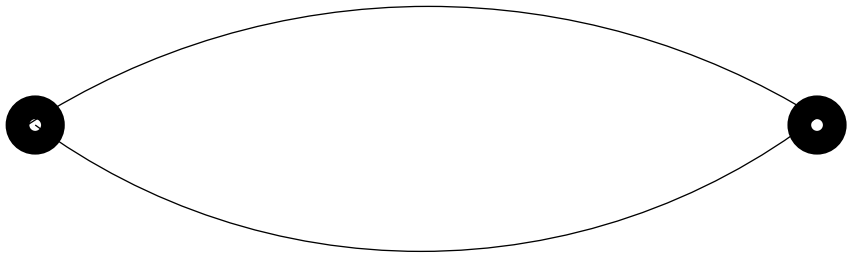
Non-Relativistic Lattice Method



- correlator

$$G(\tau) = \sum_{\vec{x}} \langle 0|\phi^\dagger(\vec{x},\tau;\vec{0},0)\phi(\vec{x},\tau;\vec{0},0)|0\rangle \quad (1)$$

$$= \sum_n e^{-E_n\tau}|\langle 0|\phi(0)|n\rangle|^2 \quad (2)$$

- if the states are well defined stationary states,

$$\rightarrow G(\tau) \sim a_0 e^{-E_0\tau} + a_1 e^{-E_1\tau} + a_2 e^{-E_2\tau} + \cdots \quad (3)$$

Non-Relativistic Lattice Method

- Non-relativistic QCD for stop

$$G(\vec{x}, t=0) = S(x) \tag{1}$$

$$G(\vec{x}, t+1) = \left[1 + \frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x}, t) \left[1 + \frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n G(\vec{x}, t) \tag{2}$$

where $S(x)$ is the source and

$$\begin{aligned} H &= -\frac{\mathbf{D}^2}{2M} \\ \delta H &= -\frac{(\vec{D}^{(2)})^2}{8(M)^3} + \frac{ig}{8(M)^2}(\vec{D}\cdot\vec{E} - \vec{E}\cdot\vec{D}) \\ &+ \frac{a^2\vec{D}^{(4)}}{24M} - \frac{a(\vec{D}^{(2)})^2}{16nM^2} \end{aligned} \tag{3}$$

Non-Relativistic Lattice Method

• relation between sNRQCD matrix elements and the stoponium wave-functions

$$|R_{1S}(\Lambda)|^2 = \frac{4\pi}{N_c}|\langle 0|\chi^\dagger\psi(\Lambda)|1S\rangle|^2 \tag{1}$$

and

$$|R_{1P}(\Lambda)|^2 = \frac{4\pi}{3N_c}|\langle 0|\chi^\dagger\overleftrightarrow{\nabla}_i\psi(\Lambda)|1P\rangle|^2 \tag{2}$$

• S-wave radial wave function computedin in nonrelativistic potential model can be interpreted as a phenomenological estimate of the above relation

Non-Relativistic Lattice Method

- $16^3 \times 256$ "quenched" lattices

- for stoponium, confinement is not important (size is smaller than ~ 1 fm)

- $\beta = 6/g^2 = 8.751$ ($a^{-1} = 50$ GeV, from S.K. D.K. Sinclair, PRD48 4408 (1993))

- $\mathcal{O}(v^2)$ NR lagrangian

- NR-related unknown energy shift E_0 and mass renormalization : mean field approximation

$$M_n = 2(Z_M M - E_0) + E_n \tag{1}$$

Non-Relativistic Lattice Method

- energy shift :

$$E_0 = -a^{-1} \ln \left[u_0 (1 - a h_0/(2n))^{2n} \right] \tag{1}$$

- mass renormalization :

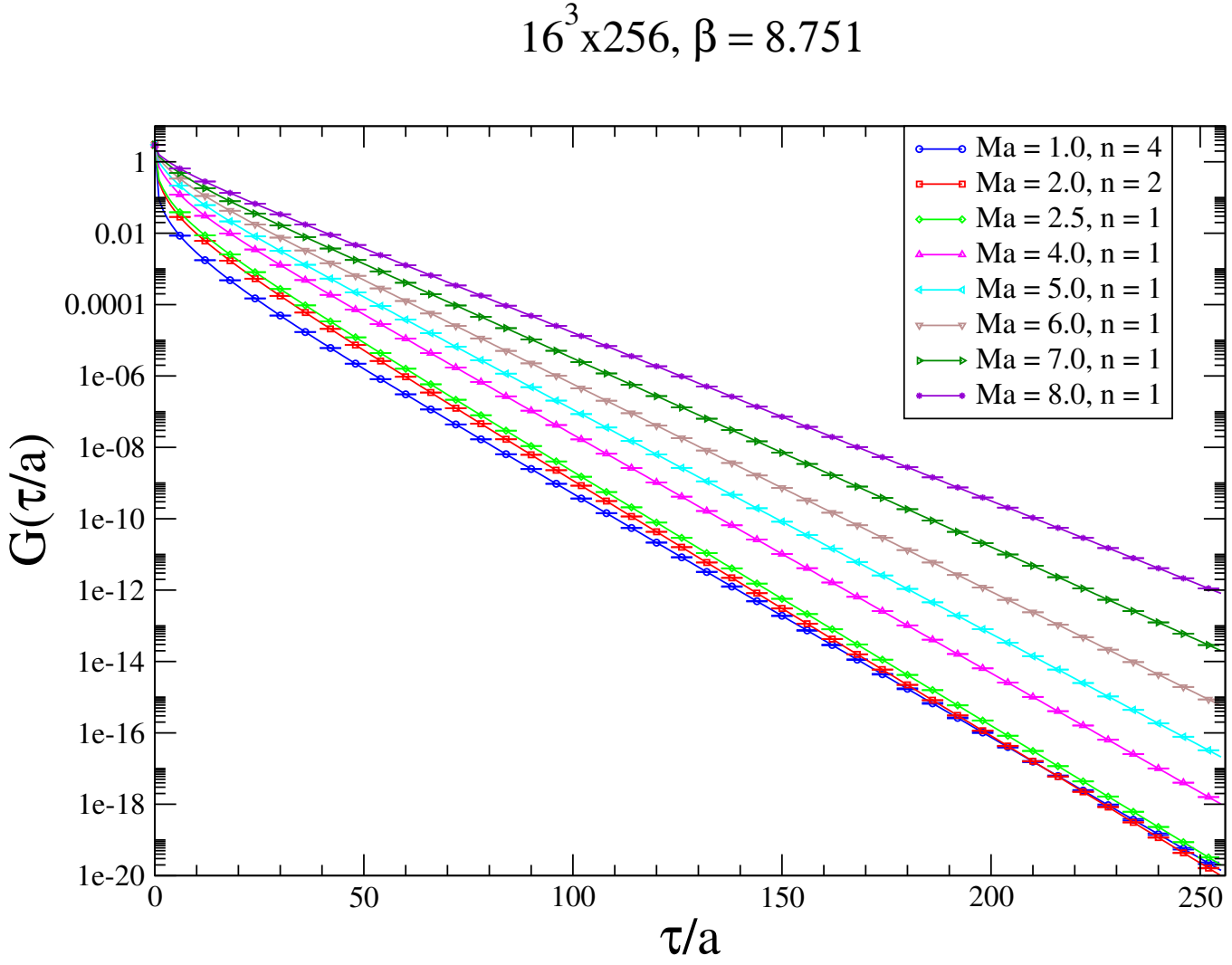
$$Z_M = u_0^{-1} (1 - a h_0/(2n)) \tag{2}$$

where $u_0 (= \langle 0 | \frac{1}{3} \mathrm{Tr} U_{plaq} | 0 \rangle^{1/4})$ is the tadpole improvement factor, and

$h_0 = 3(1 - u_0)/Ma^2$

Lattice Result

- $G(\tau)$ for S-wave stoponium



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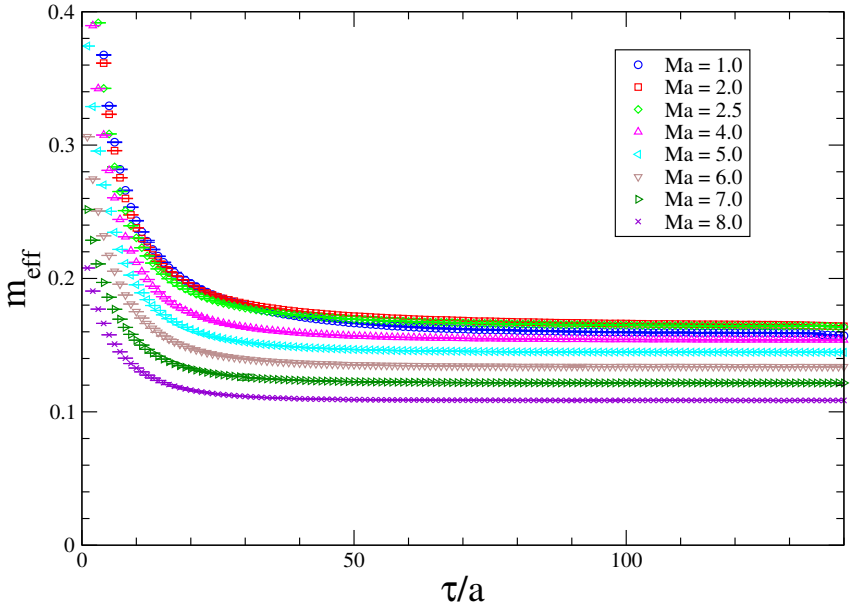
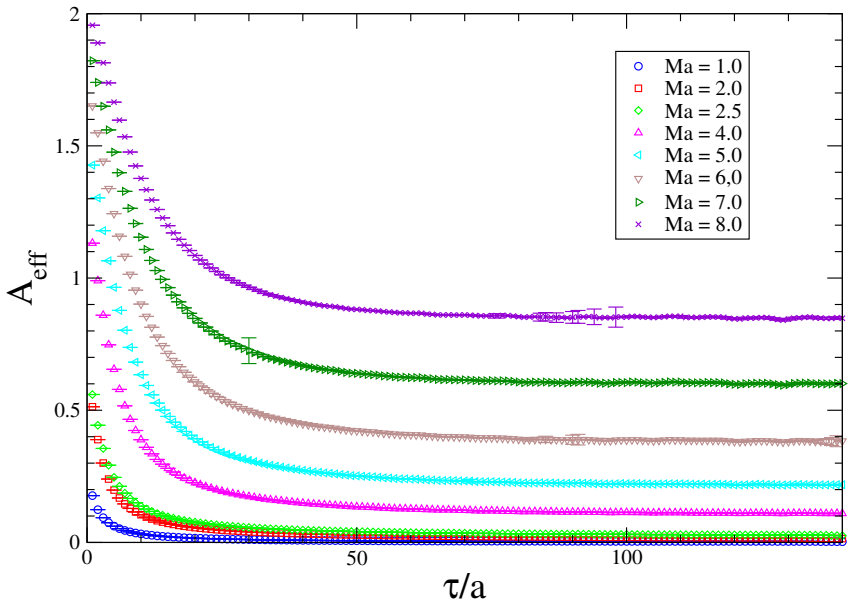




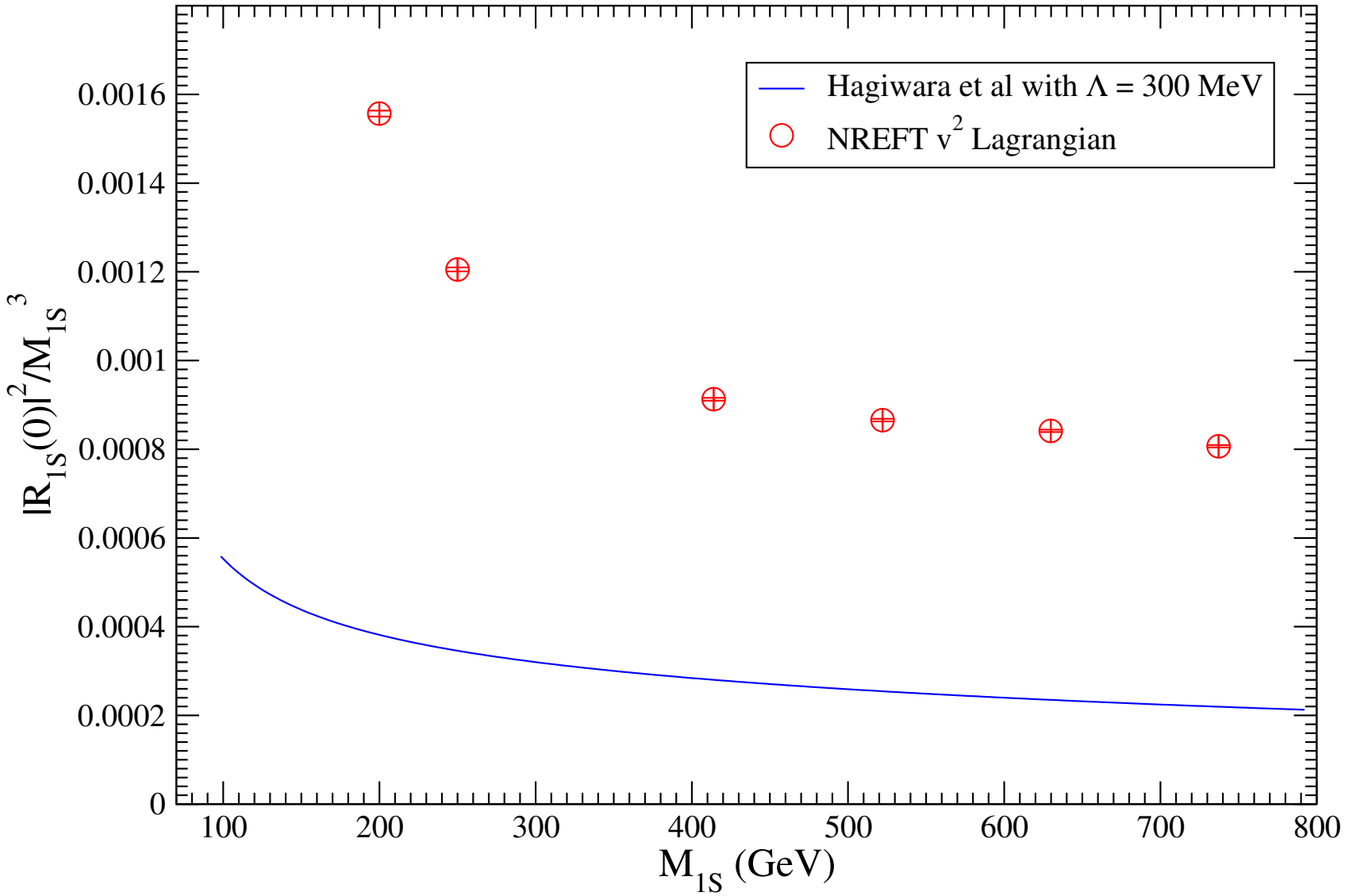
Lattice Result

Ma	Z_M	$E_0 a$	$2(Z_M M - E_0)a$	$E_{1S}a$	$\lvert R_{1S}(0)\rvert^2 a^{-3/2}$
1.0	1.047	0.2852	1.524	0.1619(2)	0.00507(4)
2.0	1.047	0.1788	3.830	0.1688(1)	0.02377(6)
2.5	1.030	0.1583	4.834	0.1671(1)	0.03599(7)
4.0	1.047	0.1257	8.125	0.1553(1)	0.1237(2)
5.0	1.053	0.1149	10.30	0.1455(1)	0.2353(3)
6.0	1.056	0.1078	12.46	0.1344(1)	0.4014(4)
7.0	1.059	0.1027	14.62	0.1220(1)	0.6173(5)
8.0	1.061	0.0989	16.78	0.1087(1)	0.8613(6)

Ma	M_{1S} (GeV)	$\lvert R_{1S}(0)\rvert^2/M_{1S}^3[\times 10^{-4}]$
1.0	84.28(1)	4.43(5)
2.0	200.0(1)	1.557(7)
2.5	250.0(1)	1.205(5)
4.0	414.0(1)	9.13(3)
5.0	522.1(1)	8.66(3)
6.0	629.8(1)	8.41(3)
7.0	737.2(1)	8.07(3)
8.0	844.4(1)	7.49(3)

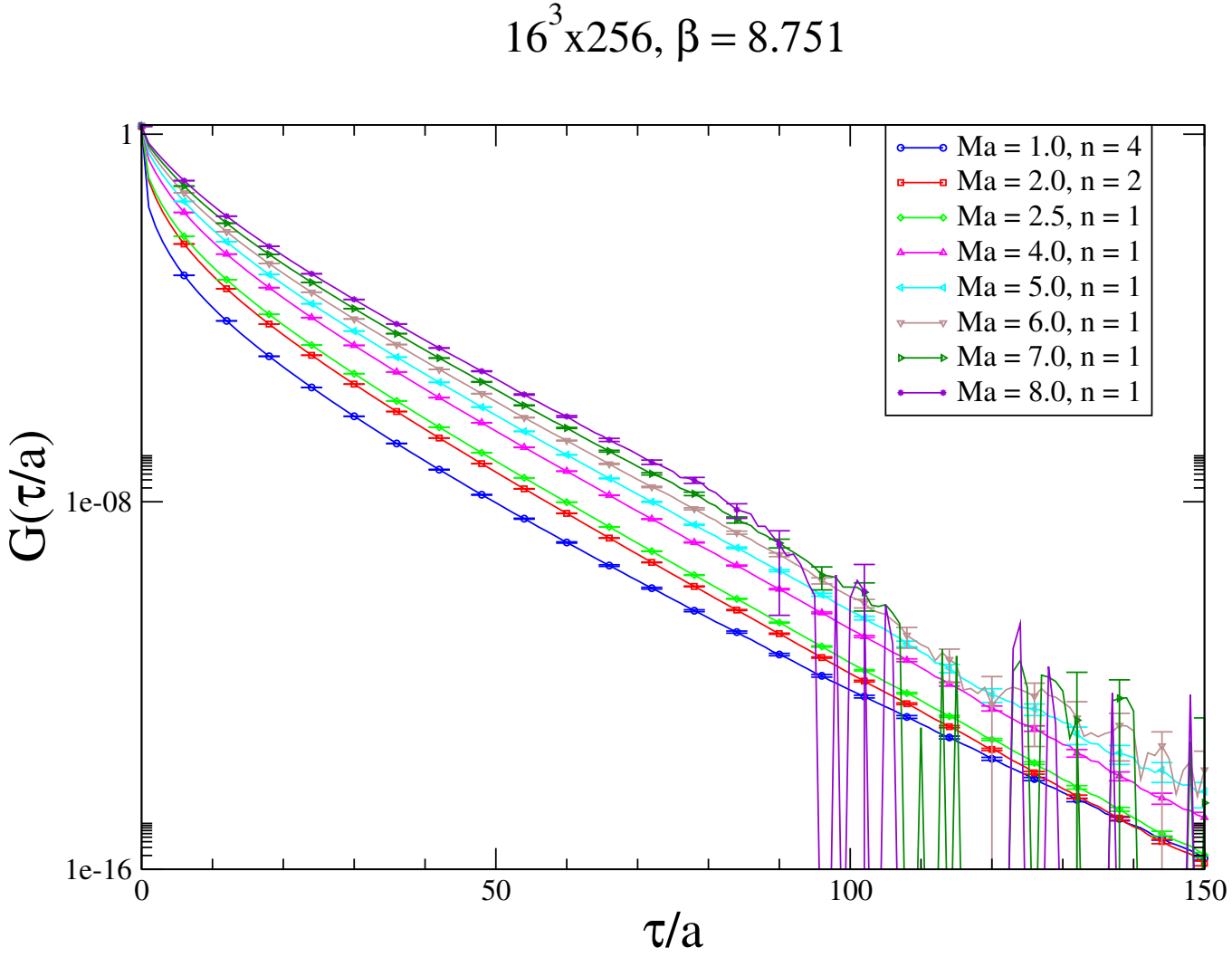
Lattice Result

- $|R_{1S}(0)|^2/M_{1S}^3$ (cf. Hagiwara et al, NPB344 (1990) 1)



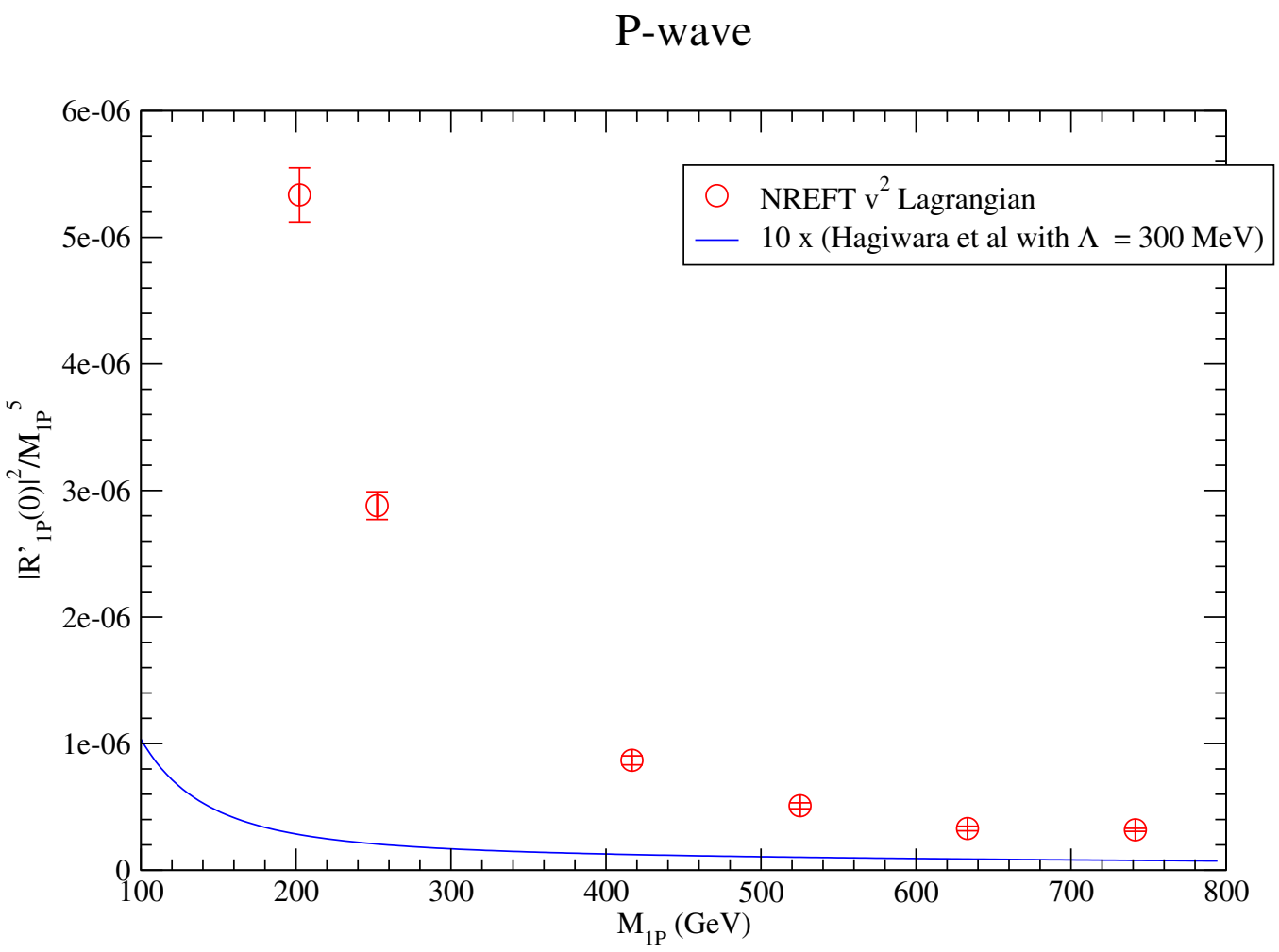
Lattice Result

- $G(\tau)$ for P-wave stoponium



Lattice Result

- $|R'_{1P}(0)|^2/M^4_{1P}$ (cf. Hagiwara et al, NPB344 (1990) 1)



Conclusion

- stoponium, bound states of heavy scalar quark and anti-scalar quark, which are binded by the strong interaction are studied with lattice gauge theory method

- $\frac{|R_{1S}(0)|^2}{M_{1S}^3}$: factor 3.5 $\sim$ 4 larger than a potential model result

- $\frac{|R'_{1P}(0)|^2}{M_{1P}^4}$: factor 33 $\sim$ 190 larger than a potential model result

- further study on systematic errors is necessary

  quenched approximation $\rightarrow$ dynamical quarks

  mean field approximation

  relativistic correction

  radiative correction