Threshold production of top-quark pairs at N^{3}LO

Jan Piclum

based on:

M. Beneke, Y. Kiyo, P. Marquard, A. Penin, JP, M. Steinhauser
PRL 115 (2015) 192001 [arXiv:1506.06864]

M. Beneke, A. Maier, JP, T. Rauh NPB 899 (2015) 180 [arXiv:1506.06865]

M. Beneke, Y. Kiyo, A. Maier, JP arXiv:1605.03010

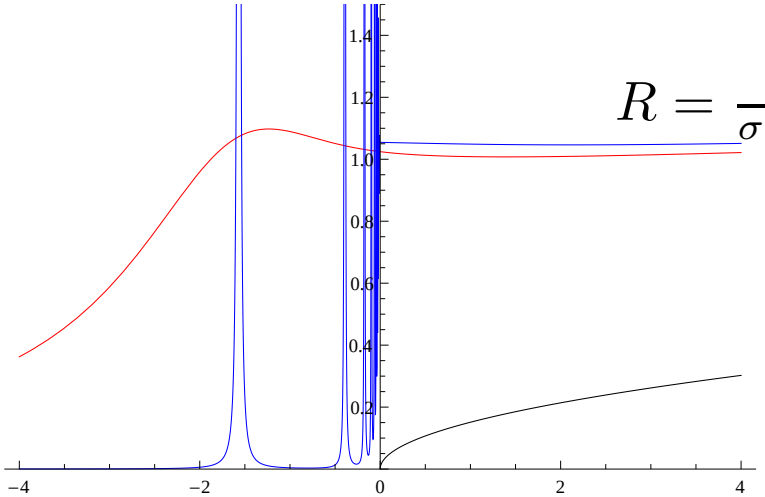
Threshold Production

relative velocity of quark-antiquark pair is small:

$$\sqrt{s} = E + 2m_Q \approx 2m_Q \quad \Rightarrow \quad v = \sqrt{\frac{E}{m_Q}} \ll 1\,; \quad v \sim \alpha_s(m_Q v)$$

- multi-scale problem: mass m_Q, momentum $m_Q v$, energy $m_Q v^2$
- perturbation theory breaks down due to terms proportional to $\frac{\alpha_s}{v}$
  $\rightsquigarrow$ Coulomb resummation

- $b\bar{b}$: bound-state resonances
- $t\bar{t}$: large width prevents existence of bound states



$$R = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

E [GeV]

Effective Theory Setup

[Thacker, Lepage; Lepage, Magnea, Nakhleh, Magnea, Hornbostel; Bodwin, Braaten, Lepage]
[Pineda, Soto; Beneke, Signer, Smirnov; Brambilla, Pineda, Soto, Vairo]

scale hierarchy: $m_t \gg m_t v \gg m_t v^2 \gg \Lambda_{\mathrm{QCD}}$

QCD full theory

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QCD — full theory

$\downarrow$ integrate out hard modes: $k^0 \sim k^i \sim m_t$
hard subgraphs become point-like vertices
$\rightsquigarrow$ hard matching coefficients

NRQCD

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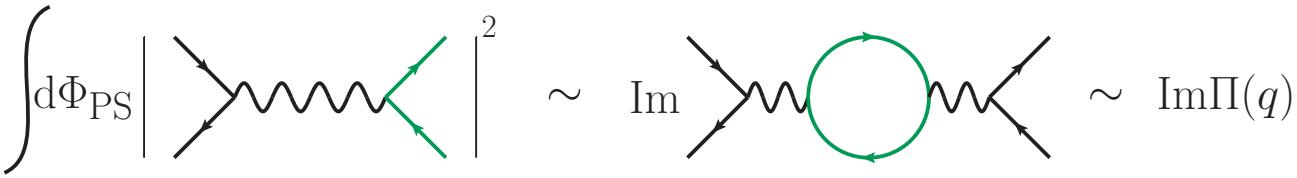
NRQCD

$\downarrow$ integrate out soft modes: $k^0 \sim k^i \sim m_t v$
soft subgraphs become instantaneous, non-local vertices
$\rightsquigarrow$ potentials

pNRQCD — contains potential quarks and ultrasoft gluons

Calculating the Cross Section

use optical theorem:



compute polarisation function in pNRQCD: $\Pi(q) \sim c_v^2\, G(\vec{0}, \vec{0}; E)$

- compute hard matching coefficients
- compute potential
- solve Schrödinger equation
- compute ultrasoft corrections

power counting:

$$\sum_n \left(\frac{\alpha_s}{v}\right)^n \times \Big\{1;\ \underbrace{\alpha_s, v}_{\text{NLO}};\ \underbrace{\alpha_s^2, \alpha_s v, v^2}_{\text{N}^2\text{LO}};\ \underbrace{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3}_{\text{N}^3\text{LO}};\ \ldots\Big\}$$

Theory Status

- N^2LO [Hoang, Teubner; Melnikov, Yelkhovsky; Yakovlev; Beneke, Signer, Smirnov; Nagano, Ota, Sumino; Penin, Pivovarov]

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  - potential contributions [Beneke, Kiyo, Schuller; Kniehl, Penin, Smirnov, Steinhauser]
  - 3-loop static potential [Anzai, Kiyo, Sumino; Smirnov, Smirnov, Steinhauser]
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- finite-width effects [Fadin, Khoze; Hoang, Reißer, Ruiz-Femenía; Beneke, Jantzen, Ruiz-Femenía; Penin, JP]
- Higgs contribution [Harlander, Jeżabek, Kühn; Eiras, Steinhauser; Beneke, Maier, JP, Rauh]

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$\rightarrow$ this talk

Corrections Beyond QCD

consider $\alpha_s^2 \sim y_t^2 \sim \alpha$

- NLO QED Coulomb potential (with multiple insertions)
- NLO non-resonant correction [Beneke, Jantzen, Ruiz-Femenía]
- N^{2}LO electroweak corrections to matching coefficients [Grządkowski, Kühn, Krawczyk, Stuart; Guth, Kühn; Hoang, Reißer; Kiyo]
- N^{2}LO and N^{3}LO Higgs corrections to c_v and potential [Eiras, Steinhauser; Beneke, Maier, JP, Rauh]

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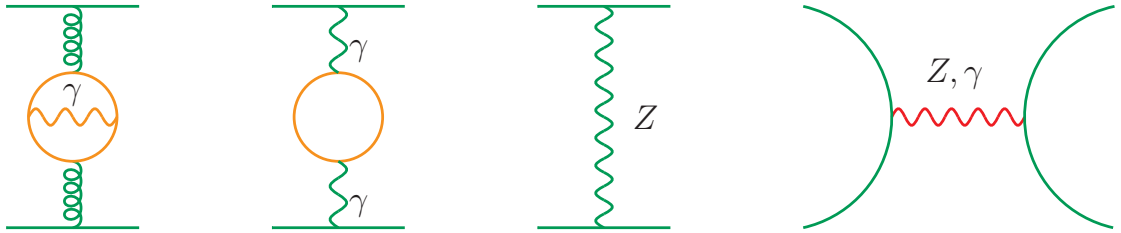
- N^{2}LO and N^{3}LO Higgs corrections to c_v and potential [Eiras, Steinhauser; Beneke, Maier, JP, Rauh]

not (yet) included:

- N^{2}LO non-resonant correction
- N^{3}LO mixed QCD/electroweak corrections to matching coefficients
- N^{3}LO electroweak corrections to potentials



QQbar_threshold

all corrections are implemented in the public program QQbar_threshold

https://qqbarthreshold.hepforge.org/

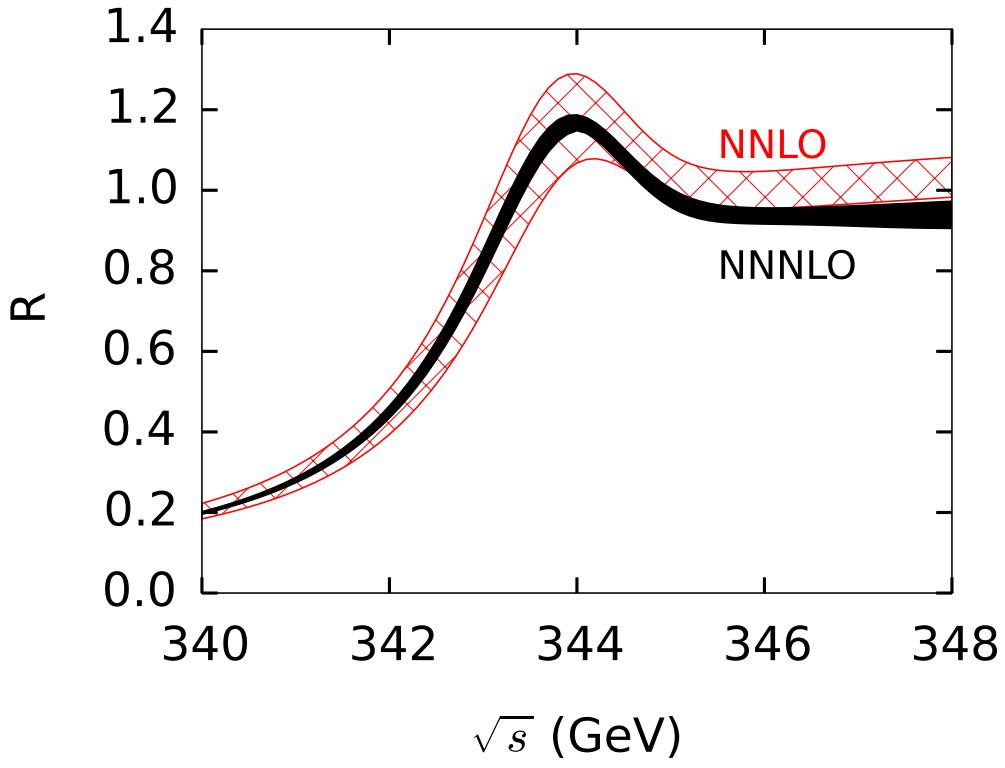
- C++ library
- Mathematica interface
- numerous options for detailed analyses
- helper functions for mass-scheme conversions and width

```
Needs["QQbarThreshold`"];
LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
Plot[
   TTbarXSection[
      sqrts, {80., 350.}, {171.5, 1.33},
      "N3LO"
   ],
   {sqrts, 340, 348}
]
```

```
#include <iostream>
#include "QQbar_threshold/QQbar_threshold.hpp"
using namespace QQbar_threshold;
int main(){
  load_grid(grid_directory() + "ttbar_grid.tsv");
  std::cout << ttbar_xsection(
     344., {80., 350.}, {171.5, 1.33}, N3LO
  ) << '\n';
}
```

[Beneke, Kiyo, Maier, JP]

Results: QCD, S-Wave Only

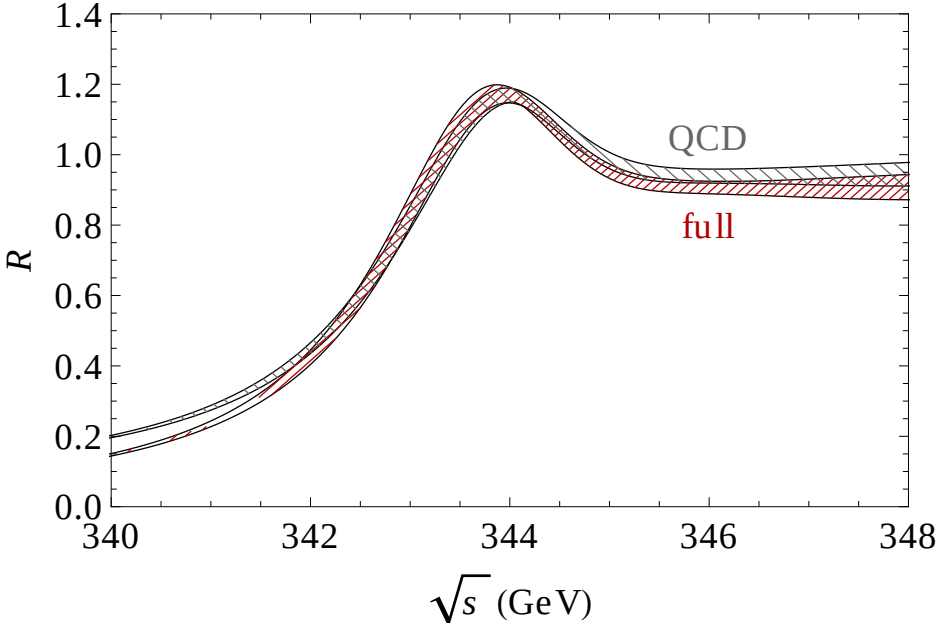


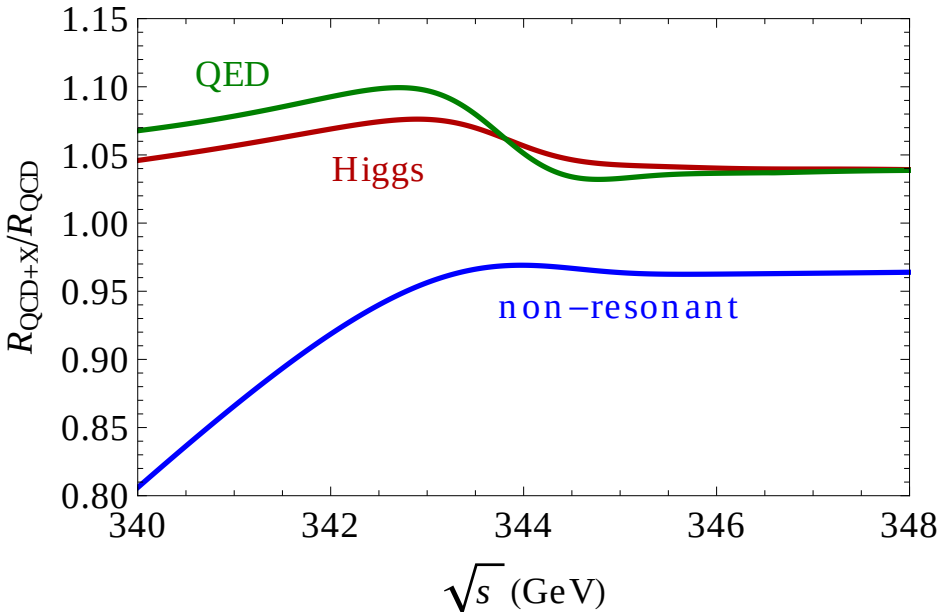
$$
\begin{aligned}
m_t^{\mathrm{PS}} &= 171.5 \text{ GeV} \\
\mu_f &= 20 \text{ GeV} \\
\Gamma_t &= 1.33 \text{ GeV} \\
\alpha_s(M_Z) &= 0.1185 \\
\mu &\in [50, 350] \text{ GeV}
\end{aligned}
$$

QCD correction under control: $\sim 3\%$ scale uncertainty

[Beneke, Kiyo, Marquard, Penin, JP, Steinhauser 2015]

Results: Corrections Beyond QCD



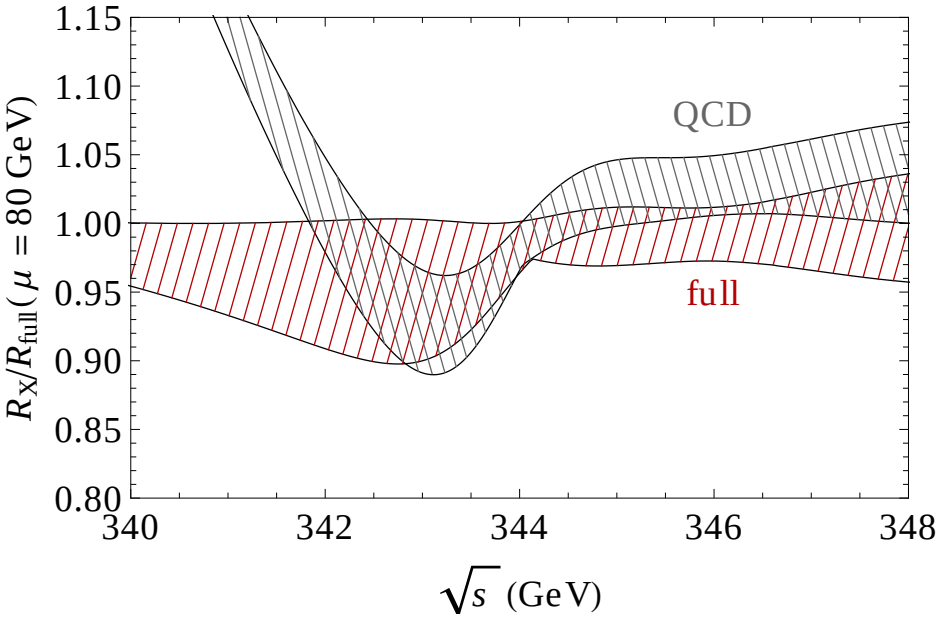


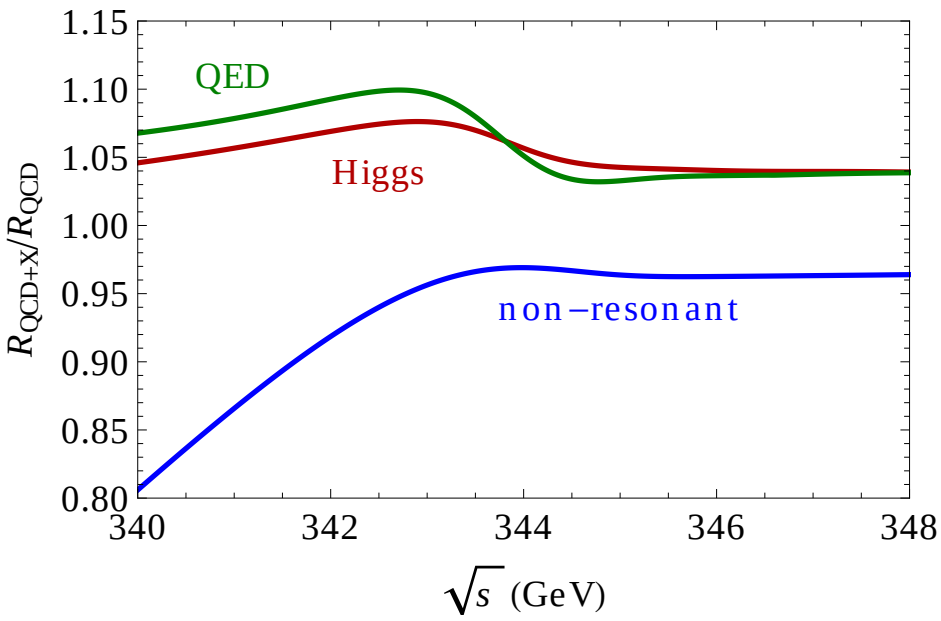
$$m_H = 125 \text{ GeV}, \quad \alpha(M_Z) = 1/128.944$$

(QCD now includes P-wave contribution)

update of [Beneke, Maier, JP, Rauh 2015]

Results: Corrections Beyond QCD





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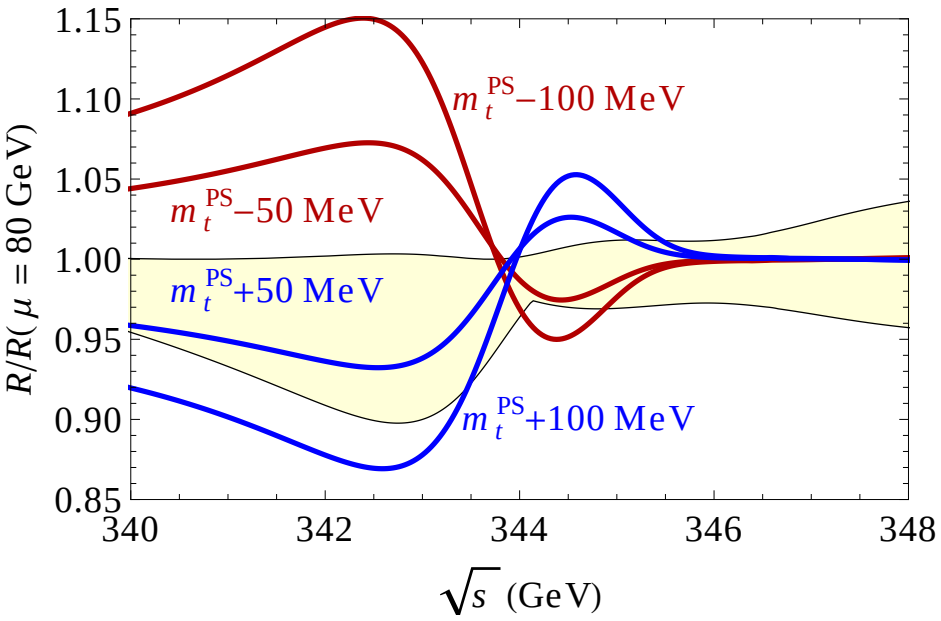
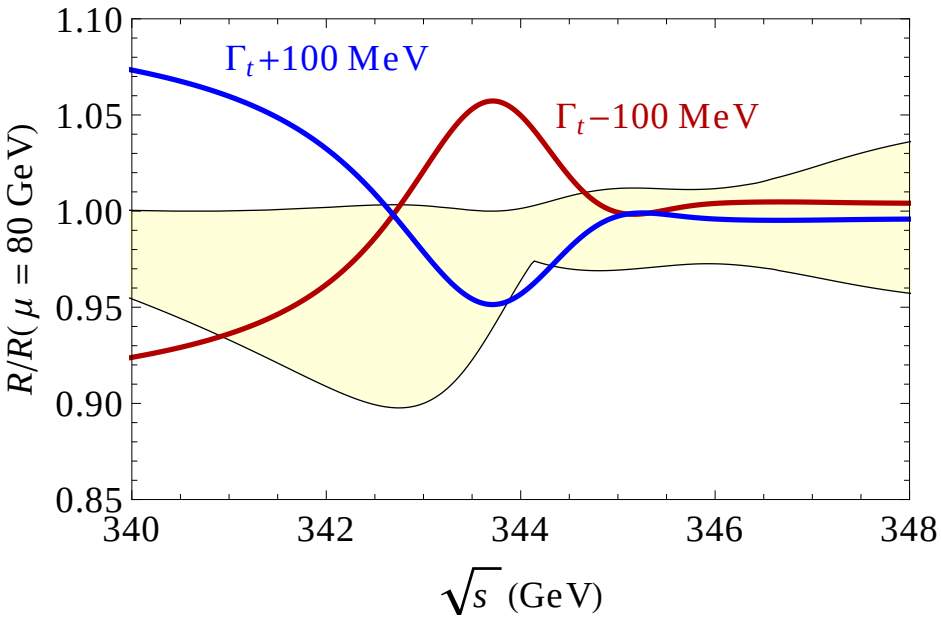
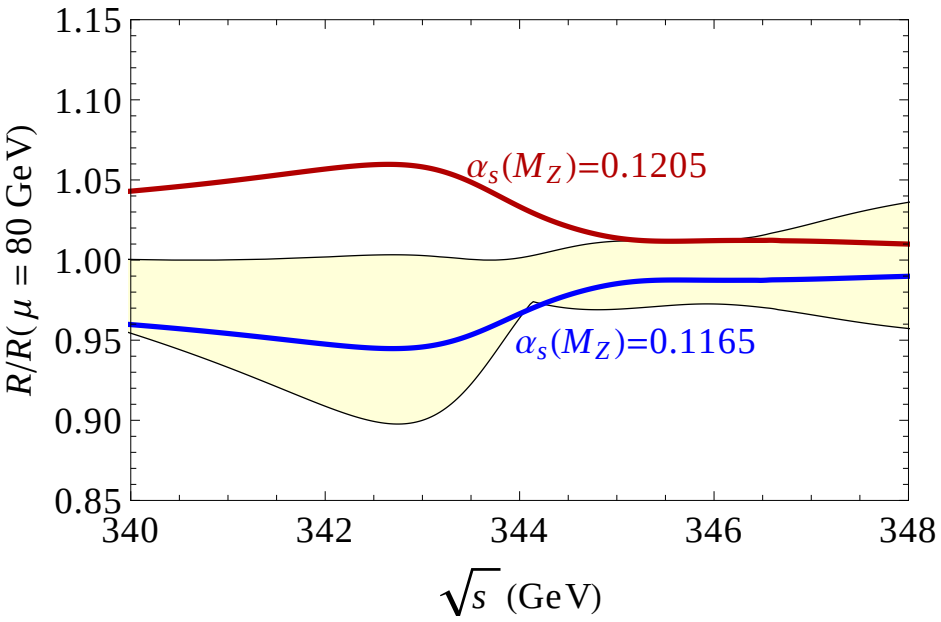
Results: Dependence on Mass and Width

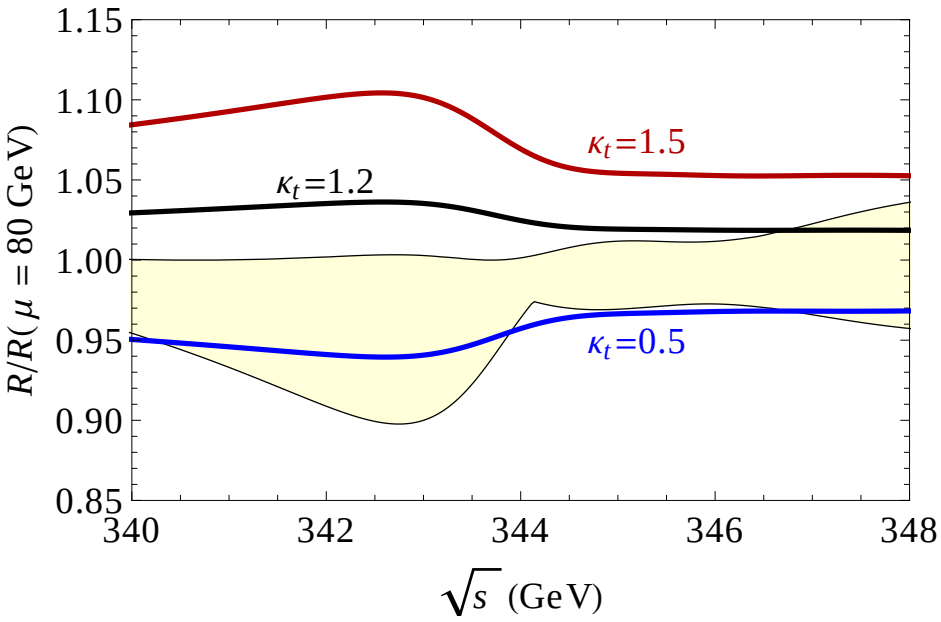




see F. Simon arXiv:1603.04764 for more detailed analysis of mass dependence, including experimental uncertainties

Results: Dependence on α_s and Yukawa Coupling





$$\kappa_t = y_t / y_t^{\mathsf{SM}}\,, \quad y_t^{\mathsf{SM}} = \frac{\sqrt{2} m_t}{v}$$

from operator

$$\Delta\mathcal{L} = -\frac{c_{\mathsf{NP}}}{\Lambda^2}(\phi^\dagger\phi)(\bar{Q}_3 i\sigma^2\phi^* t_R) + \text{h.c.}$$

update of [Beneke, Maier, JP, Rauh 2015]

Summary

- threshold production allows for very precise and theoretically clean determination of top-quark mass
- very high theoretical precision is required
- N^3LO QCD corrections are under control
- electroweak corrections are being included; effects are sizeable
- public program is available: https://qqbarthreshold.hepforge.org/

lots of work still to be done, e.g.:

- N^2LO non-resonant corrections
- combination of fixed-order and resummed calculations