The bottom quark mass from the $\Upsilon(1S)$ at NNNLO

Based on

Ayala, Cvetic, Pineda: JHEP 1409 (2014)045

Ayala, Cvetic, Pineda: ACAT2016 proceedings

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Motivation

m_Q. Fundamental parameter in heavy quarkonium physics.

Usual definitions:

- $m_{\overline{\rm MS}} \to$ short distance mass.
- $m_{\rm OS} \to$ natural definition for heavy quark physics.

$$m_{\rm OS} = m_{\overline{\rm MS}} + \sum_{n=0}^{\infty} r_n \alpha_s^{n+1} \,,$$

Renormalon (OPE) analysis predicts $r_n \sim n!$. Therefore $m_{\rm OS}$ suffers from renormalon ambiguities:

$$\delta_{np}^{(\rm pert.)} m_{\rm OS} = \delta_{np}^{(\rm pert.)} m_{\overline{\rm MS}} (1 + B_1 \alpha_s + B_2 \alpha_s^2 + \cdots) \sim \Lambda_{\rm QCD}!$$

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$$m_{\rm OS} = m_{\overline{\rm MS}} + \int_0^\infty {\rm d}t\, e^{-t/\alpha_s}\, B[m_{\rm OS}](t)\,, \qquad B[m_{\rm OS}](t) \equiv \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}.$$

The behavior of the perturbative expansion at large orders is dictated by the closest singularity to the origin of its Borel transform ($u = \frac{\beta_0 t}{4\pi}$).

$$B[m_{\rm OS}](t) = N_m \nu \frac{1}{(1-2u)^{1+b}} \left(1 + c_1(1-2u) + \cdots\right) + (\text{analytic term}),$$

$$r_n \overset{n\to\infty}{=} N_m\, \nu \left(\frac{\beta_0}{2\pi}\right)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)} \left(1 + \frac{b}{(n+b)} c_1 + \cdots\right).$$

$$b = \frac{\beta_1}{2\beta_0^2}\,, \qquad c_1 = \frac{1}{4\, b \beta_0^3}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right), \qquad \cdots$$

$$M_{\Upsilon(nS)} = 2m_{\rm OS} + E_n^{\rm OS} = 2m_{\rm OS} + A_n^{1,{\rm OS}}\alpha^2 + A_n^{2,{\rm OS}}\alpha^3 + \cdots$$

$M_{\Upsilon(nS)}$ is renormalon free $\rightarrow E_n^{\rm OS}$ has a renormalon.
At the static level: $2m_{OS} + V_s(r)$ is renormalon free.
Alternative:

$$m_{\rm RS}(\nu_f) = m_{\rm OS} - \sum_{n=0(1)}^{\infty} N_m\,\nu_f\left(\frac{\beta_0}{2\pi}\right)^n \alpha_s^{n+1}(\nu_f)\sum_{k=0}^{\infty} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}\,.$$

Expansion in $\alpha_s(\nu)$

$$m_{\rm RS}(\nu_f) = m_{\overline{\rm MS}} + \sum_{n=0}^{\infty} r_n^{\rm RS}\alpha_s^{n+1}\,,$$

where $r_n^{\rm RS} = r_n^{\rm RS}(m_{\overline{\rm MS}}, \nu, \nu_f)$ are of natural size. We now do not loose accuracy if we first obtain $m_{\rm RS}$ and later on $m_{\overline{\rm MS}}$.

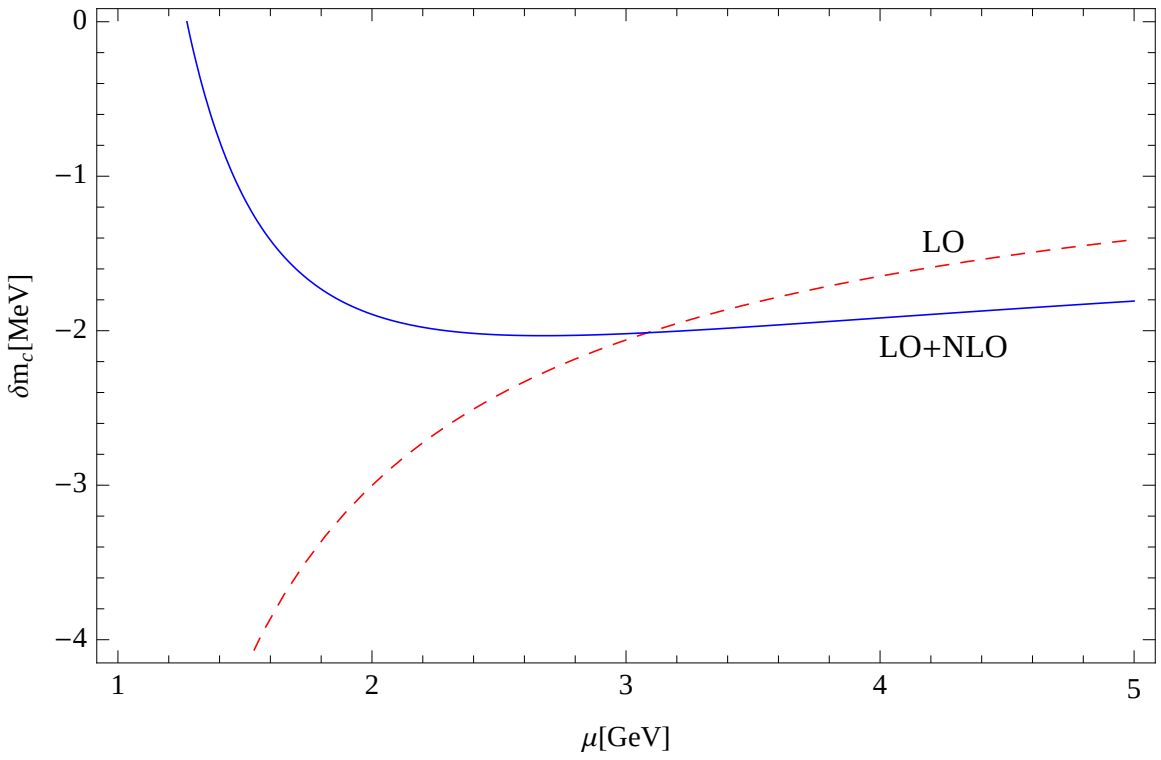
$$M_{\Upsilon(nS)} = 2m_{\rm RS} + E_n^{\rm RS} = 2m_{\rm RS} + \sum_{m=1}^{\infty} A_n^{m,{\rm RS}}\alpha_s^{m+1}\,.$$

2001 $\rightarrow$ 2016 analysis in the RS mass

- NNLO +(NNNLO logs & large β_0) $\rightarrow$ N^{3}LO
  - Static potential: Anzai et al.; Smirnov et al.
  - Heavy quarkonium mass: Penin et al.
  - Pole-$m_{\overline{\rm MS}}$ four-loop relation: Marquard et al.
- N_m: 10% $\rightarrow$ 5% error
- Light fermions: $n_f = 4 \rightarrow n_f = 3$ (for soft scale Brambilla et al.)

$n_f = 3$ light fermions in the $\overline{\mathrm{MS}}$-pole mass relation

$$m_b e^{-n} < m_c$$



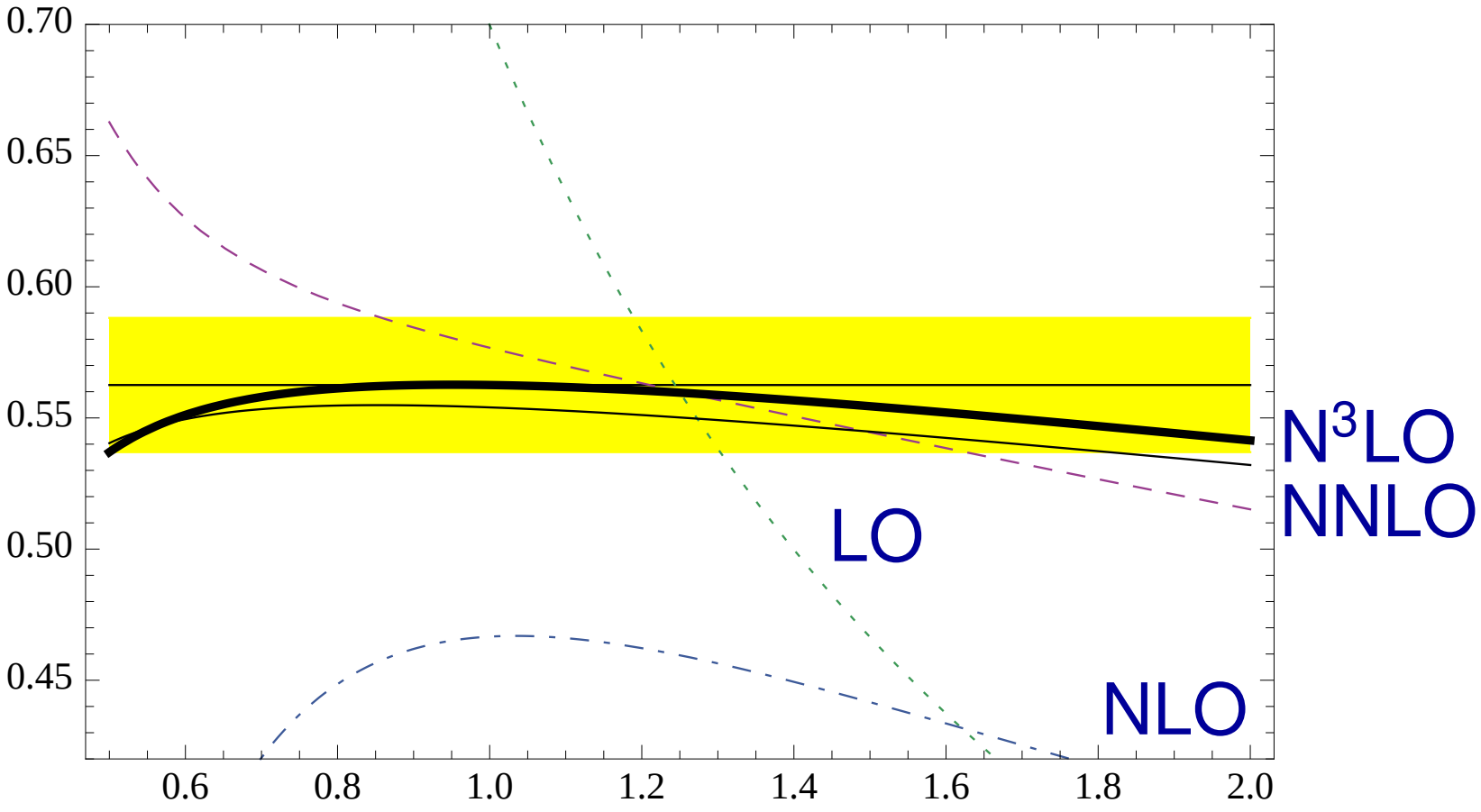
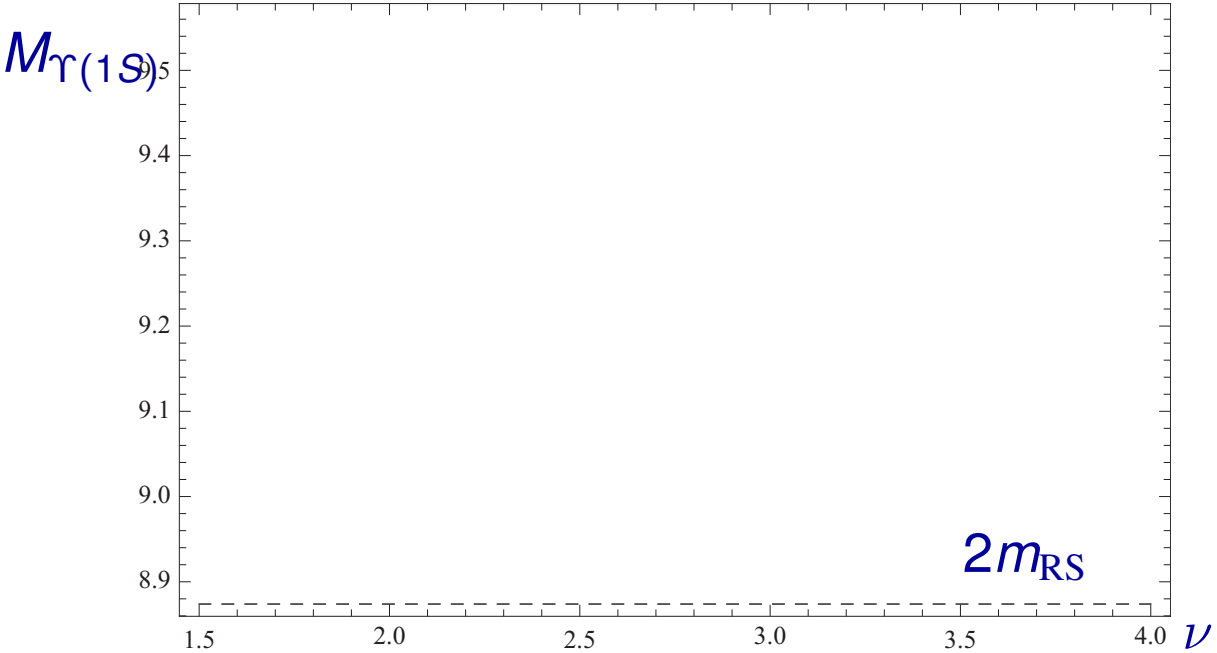
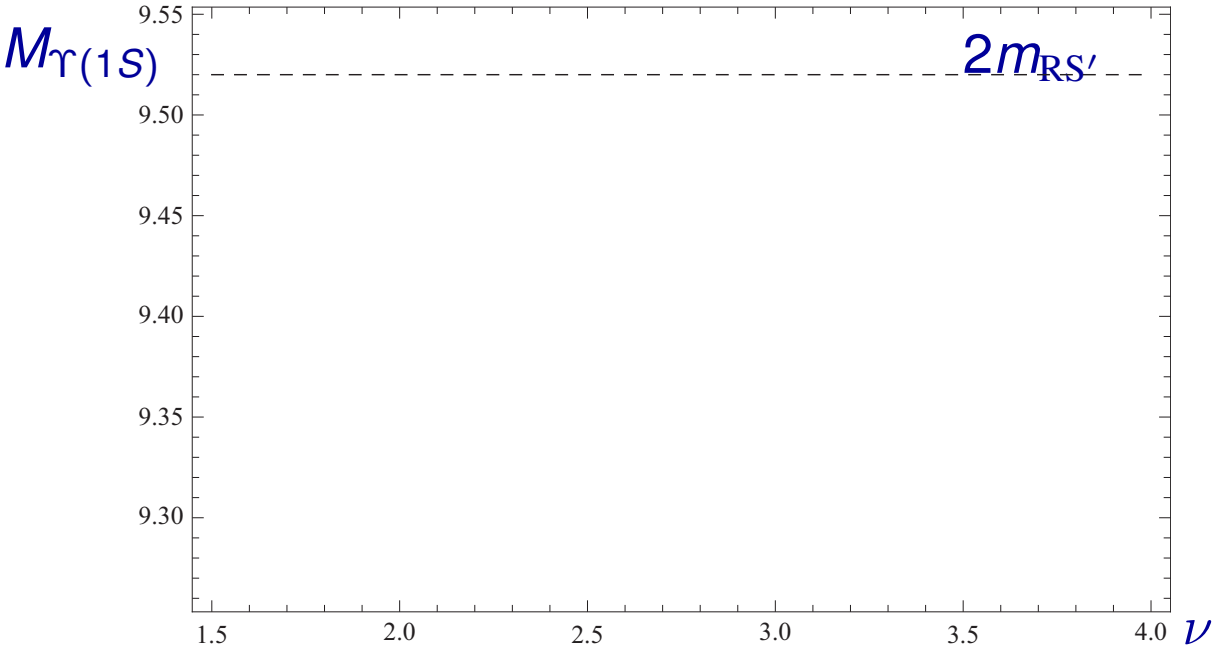


Figure: $-N_V/2 = N_m$ for $n_l = 3$, as a function of $x \equiv \nu r$, obtained from $-(N_V/2)v_n/v_n^{asym}$. v_n^{asym} is truncated at $\mathcal{O}(1/n^3)$.

$$\boxed{N_m(n_l = 0) = 0.600(29)}\,, \qquad \boxed{N_m(n_l = 3) = 0.563(26)}\,.$$

$\sim$ 20 standard deviations from zero!


$M_{\Upsilon(1S)}$
9.5
9.4
9.3
9.2
9.1
9.0
8.9
$2m_{\rm RS}$
1.5
2.0
2.5
3.0
3.5
4.0
ν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9.50
9.45
9.40
9.35
9.30
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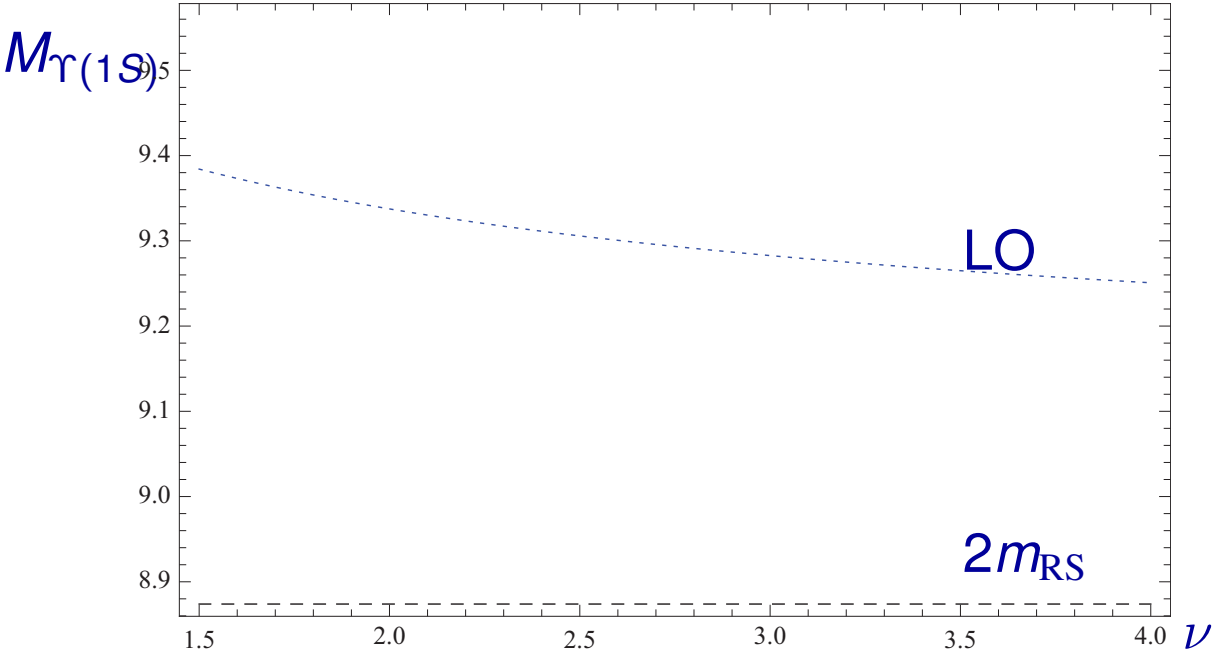
4.0
ν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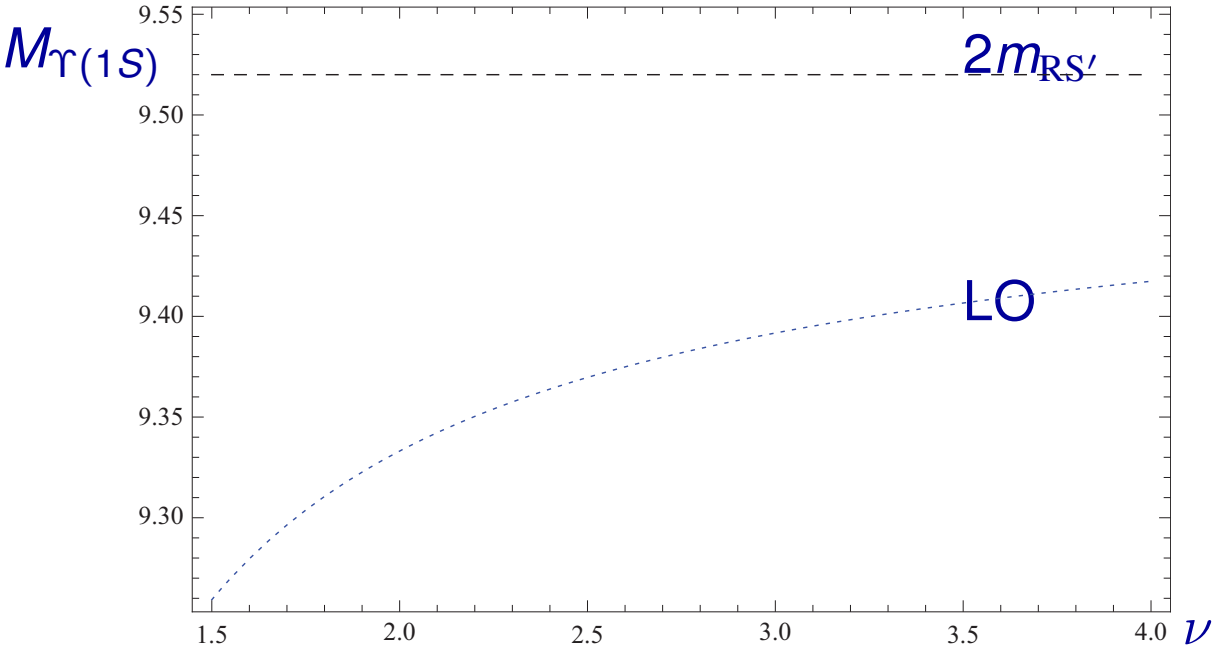
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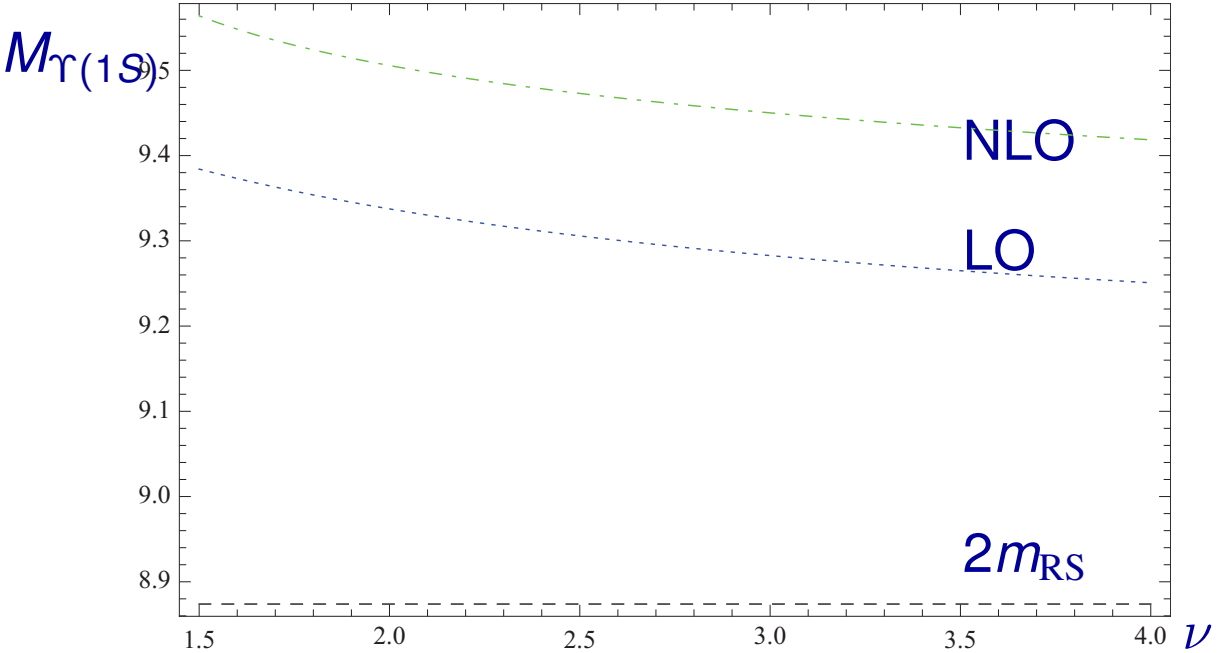
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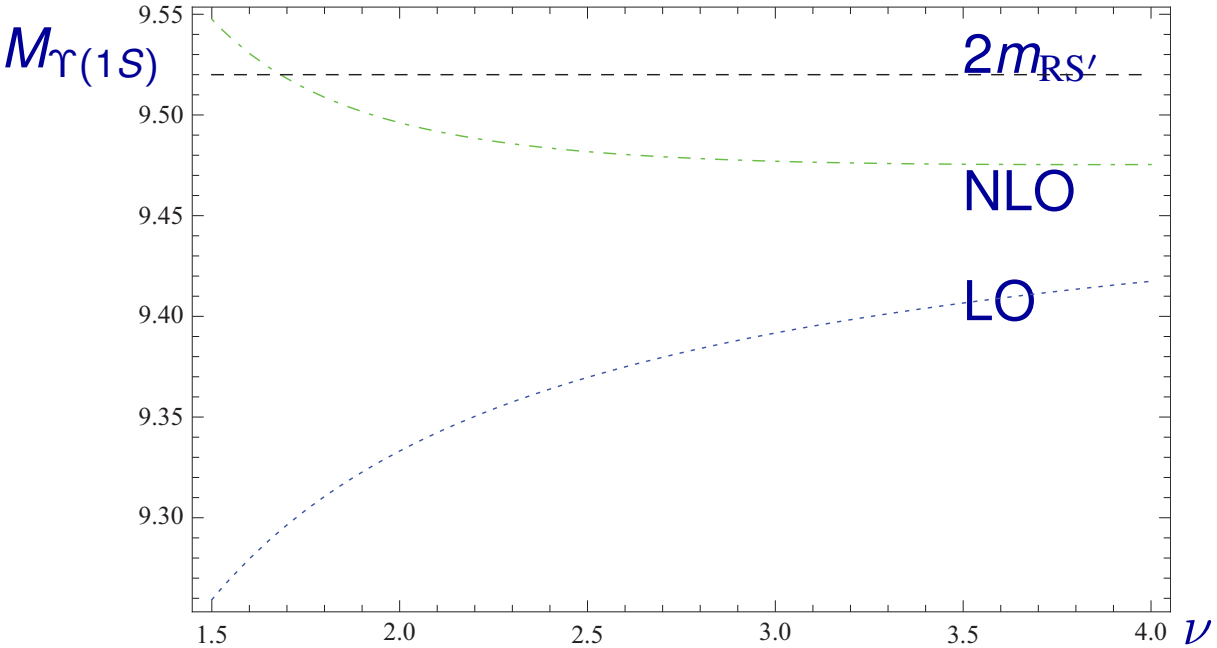
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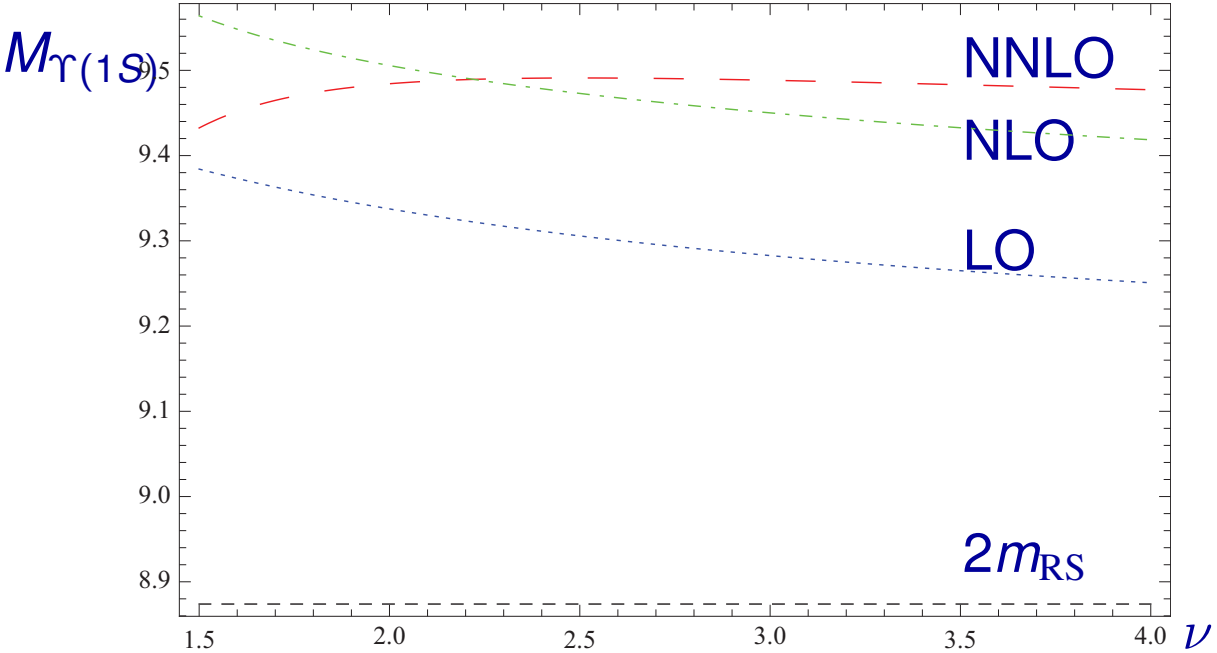
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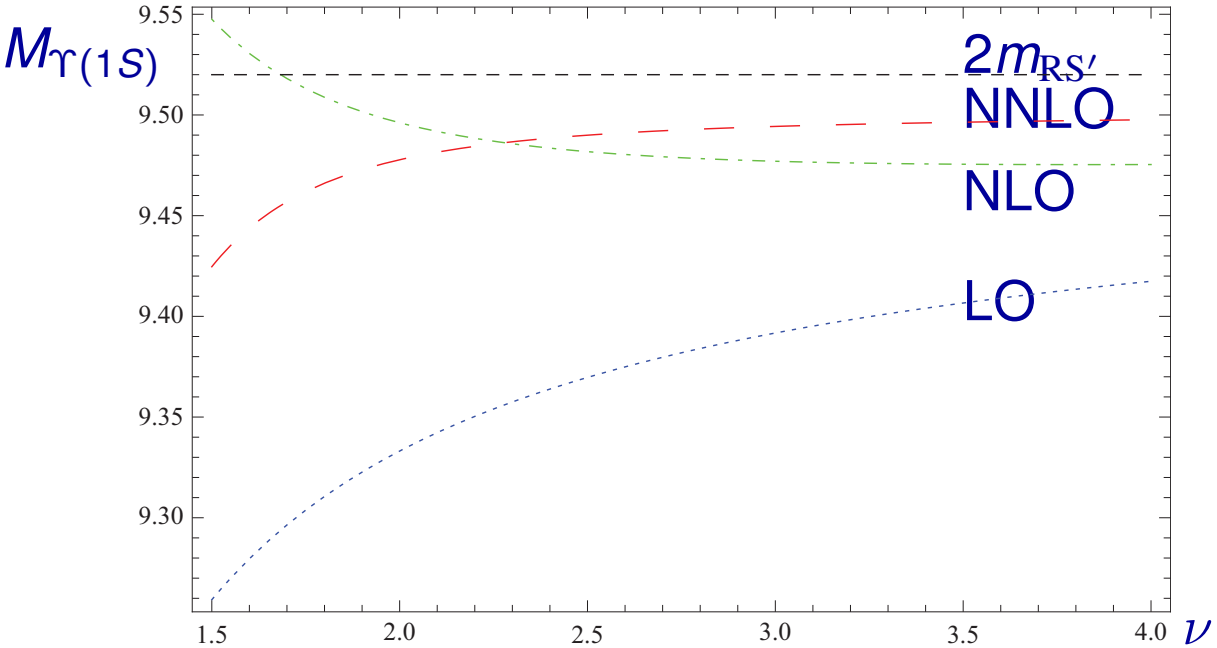
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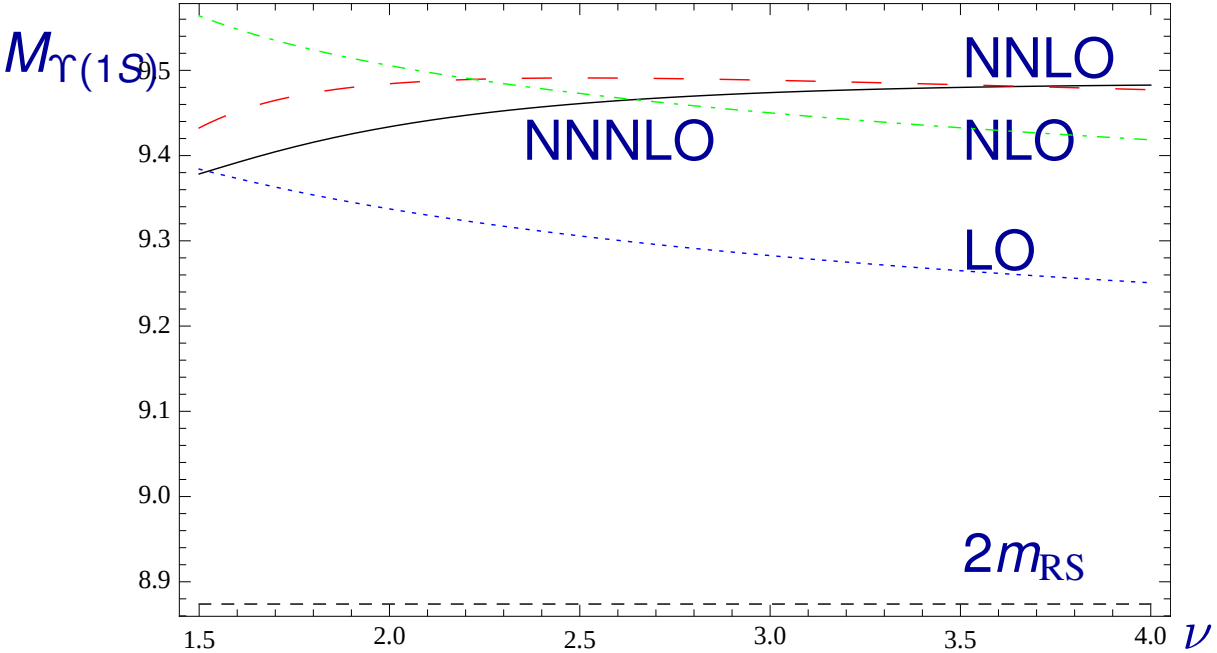
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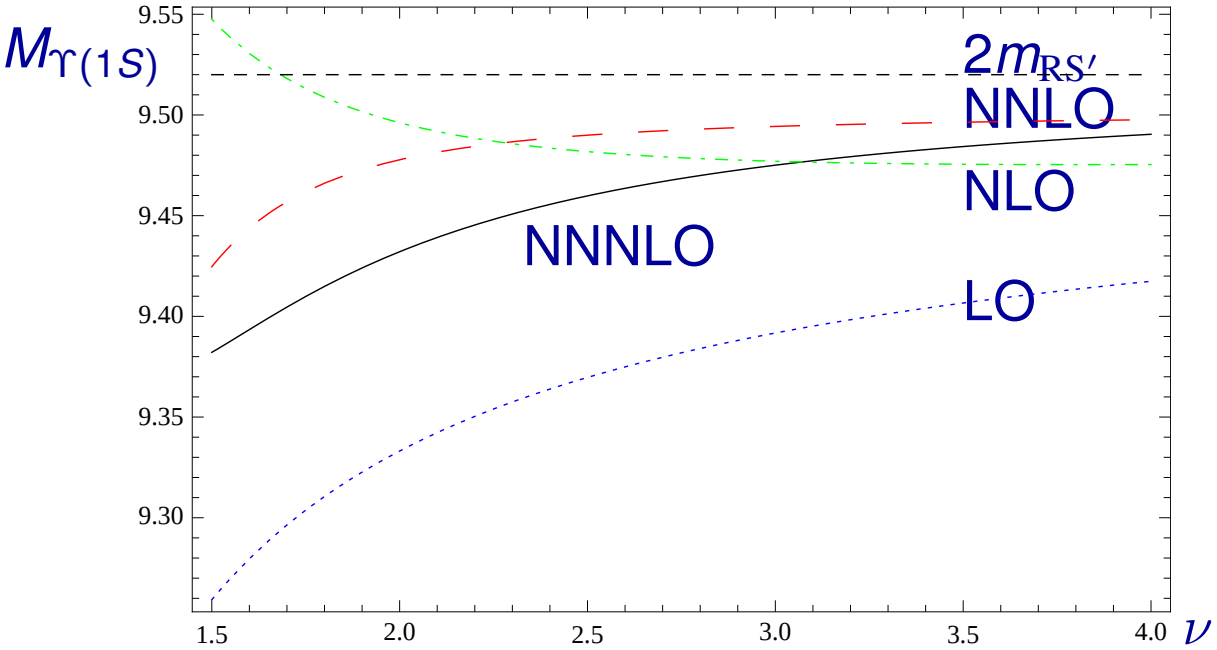
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$$
\begin{aligned}
m_{b,\mathrm{RS}}(2\mathrm{GeV}) &= 4\,437^{-11}_{+43}(\mu)^{-3}_{+5}(\nu_f)^{-2}_{+2}(\alpha_s)^{-41}_{+41}(N_m)^{-0}_{+0}(r_3);\\
\Rightarrow \quad \overline{m}_b &= 4\,217^{-10}_{+39}(\mu)^{-3}_{+5}(\nu_f)^{-5}_{+5}(\alpha_s)^{-1}_{+1}(N_m)^{-4}_{+4}(r_3).\\
m_{b,\mathrm{RS}'}(2\ \mathrm{GeV}) &= 4\,761^{-16}_{+41}(\mu)^{-3}_{+5}(\nu_f)^{+4}_{-3}(\alpha_s)^{-26}_{+26}(N_m)^{-0}_{+0}(r_3);\\
\Rightarrow \quad \overline{m}_b &= 4\,223^{-14}_{+36}(\mu)^{-2}_{+4}(\nu_f)^{-4}_{+4}(\alpha_s)^{-1}_{+1}(N_m)^{-4}_{+4}(r_3).
\end{aligned}
$$

$$\overline{m}_b = 4222(40)\ \mathrm{MeV}\ .$$

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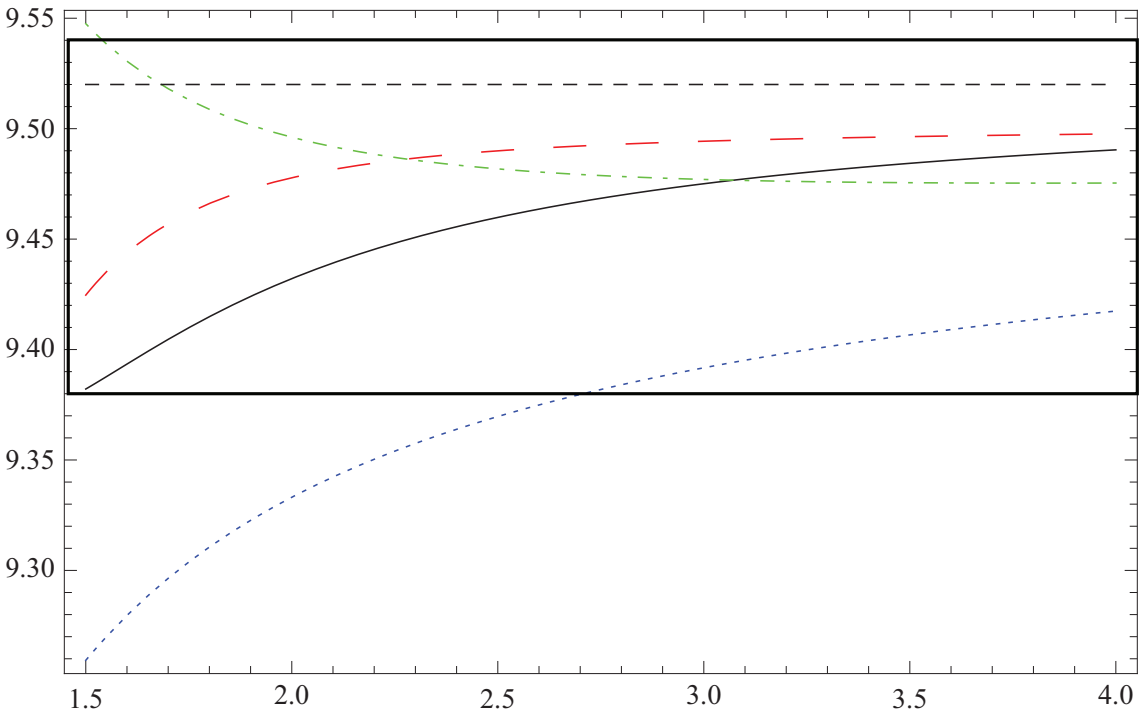

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CONCLUSIONS

Determination of the $\overline{\mathrm{MS}}$ bottom quark mass from the $\Upsilon(1S)$.

- Clean observable
- Different systematics than other determinations
- Slightly bigger number than most determinations
- Scale dependence increases from $NNLO \to N^3LO$

FUTURE I: Understand this last item

- New effects show up at N^{3}LO:
  Renormalization group improvement?
  Nonperturbative effects?
- Relate with other determinations.

FUTURE II: B_c spectrum

- New N^{3}LO result for the spectrum (Peset, Pineda & Stahlhofen)
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