Determination of m_c and m_b from quarkonium 1S energy levels in perturbative QCD

PLB 752 122 [arXiv:1510.07072]

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Collaborators:
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Yukinari Sumino (Tohoku Univ.)

2016/06/09, QWG 2016, PNNL

Plan of my talk

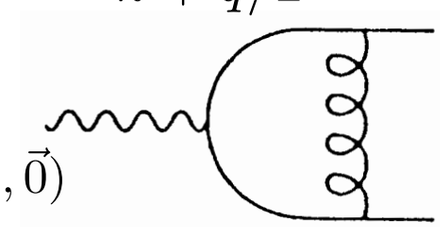
- Theoretical framework (potential-NRQCD)
- Current status of perturbative calculations
- Determination of m_b and m_c
- Uncertainty estimation
- summary

$$M_\Upsilon = 2m_b^{\rm pole} + E_{\rm bin}$$
$$= 2\overline{m}_b \left[1 + f_1\alpha_s + f_2\alpha_s^2 + f_3\alpha_s^3 + f_4\alpha_s^4 + \cdots\right]$$

construction of the potential-NRQCD

[Pineda, Soto, '98]
[Brambilla, Pineda, Soto, Vairo, '00]

	energy k_0	momentum $\mathbf{k}$
hard (H)	$\sim m$	$\sim m$
soft (S)	$\sim mv$	$\sim mv$
potential (p)	$\sim mv^2$	$\sim mv$
ultra soft (US)	$\sim mv^2$	$\sim mv^2$



$$v=\sqrt{1-\frac{4m^2}{q_0^2}}$$

full QCD

↓ integrating out hard contribution

(effects are encoded in Wilson coefficients)

NRQCD

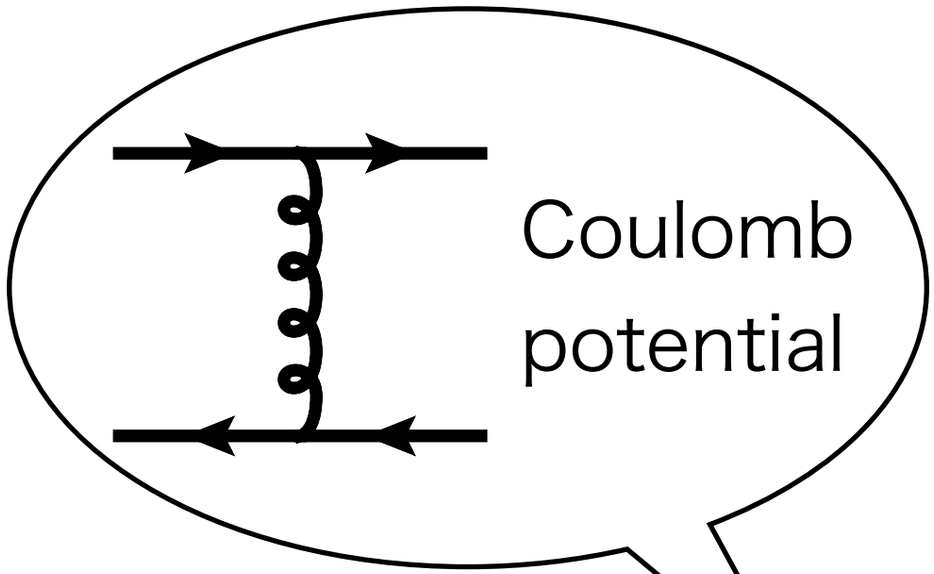
↓ integrating out soft contribution

(potential is generated)

pNRQCD

remaining degrees of freedom (DOF): heavy quark (P) ,(US)
gluon (US)
light quarks (US)

our current knowledge of the Coulomb potential



$$V_{\mathrm{C}}(r) = -\frac{C_F \alpha_S}{r}\left(v_0 + v_1 \alpha_S + v_2 \alpha_S^2 + v_3 \alpha_S^3 + \mathcal{O}(\alpha_S^4)\right)$$

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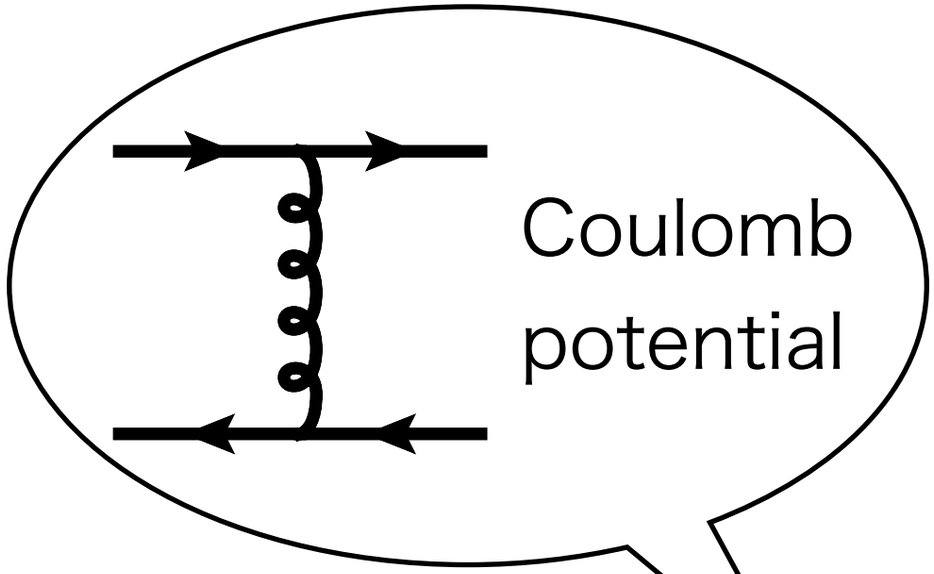

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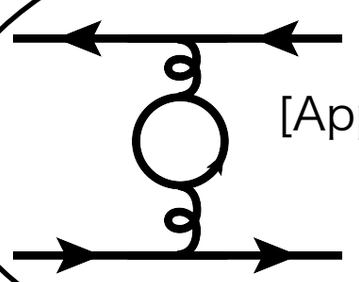


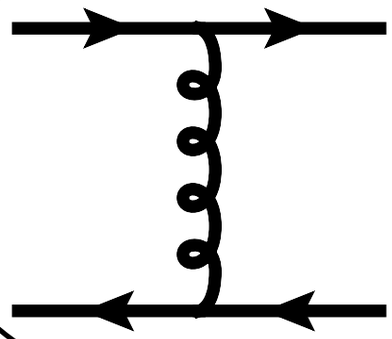
massless quark

[Appelquist, Dine, Muzinich, '77, '78]

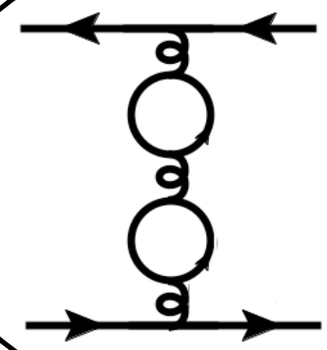
massive quark

[Fischler, '77, Billoire, '80]

our current knowledge of the Coulomb potential



Coulomb potential



massless quark

[Peter, '97, Schroder, '99]

massive quark

[Melles, '00, Hoang, '00]

[Recksigel, Sumino, '02]

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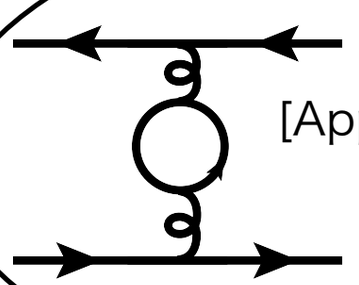

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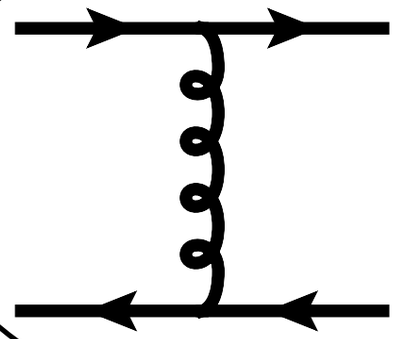
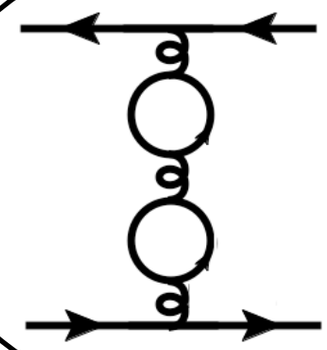


Coulomb potential



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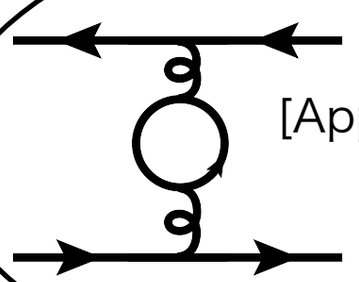

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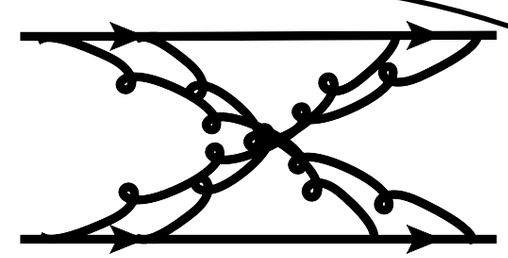
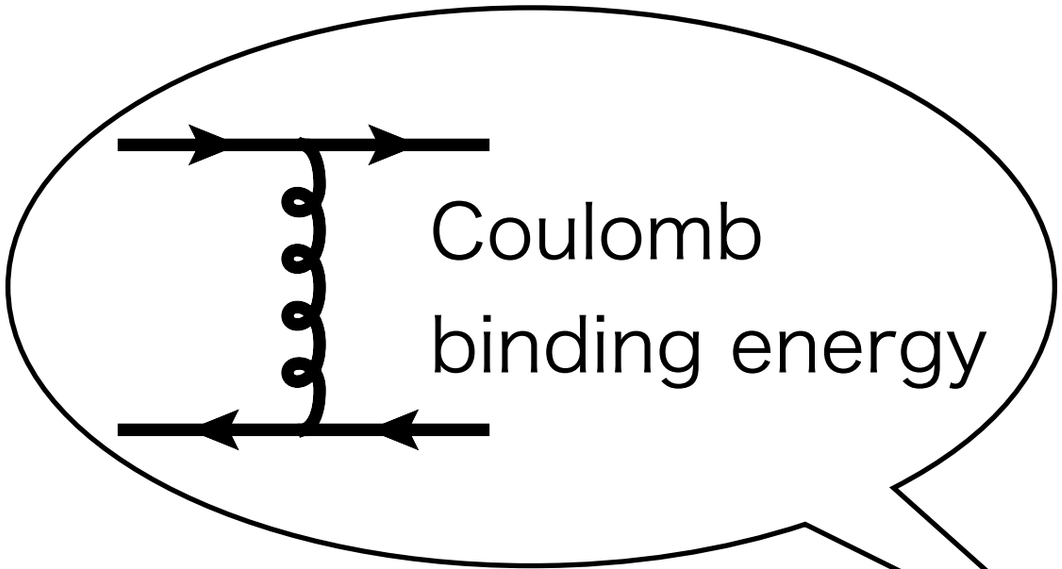


massless quark
[Anzai, Kiyo, Sumino, '09]
[Smirnov, Smirnov, Steinhauser, '09]

our current knowledge of binding energy



$$E_{\rm bin} = -\frac{4}{9} m_b^{\rm pole} \alpha_s^2 \left(P_0 + P_1 \frac{\alpha_s}{\pi} + P_2 \frac{\alpha_s^2}{\pi^2} + P_3 \frac{\alpha_s^3}{\pi^3} \right)$$

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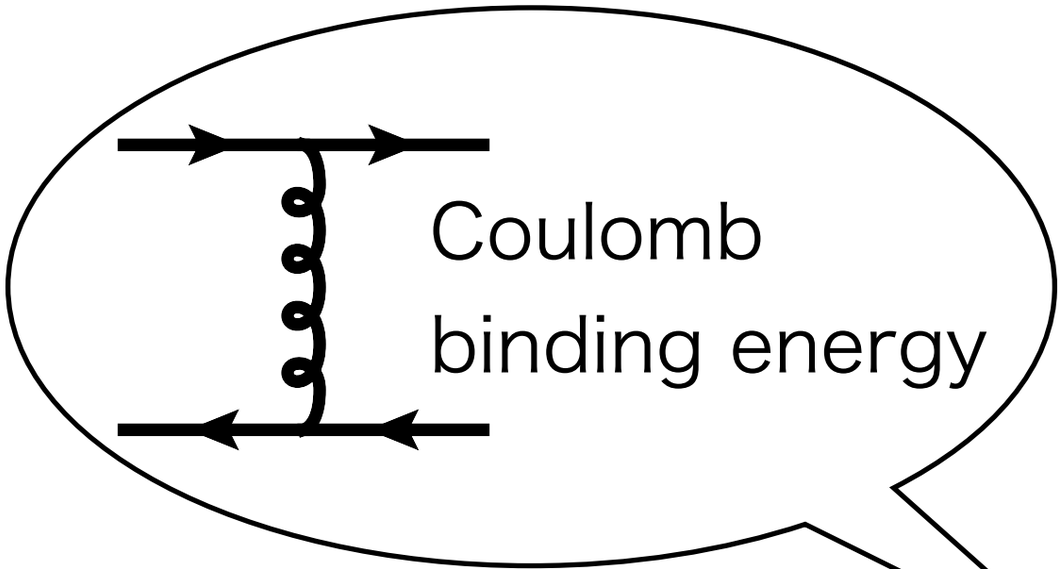

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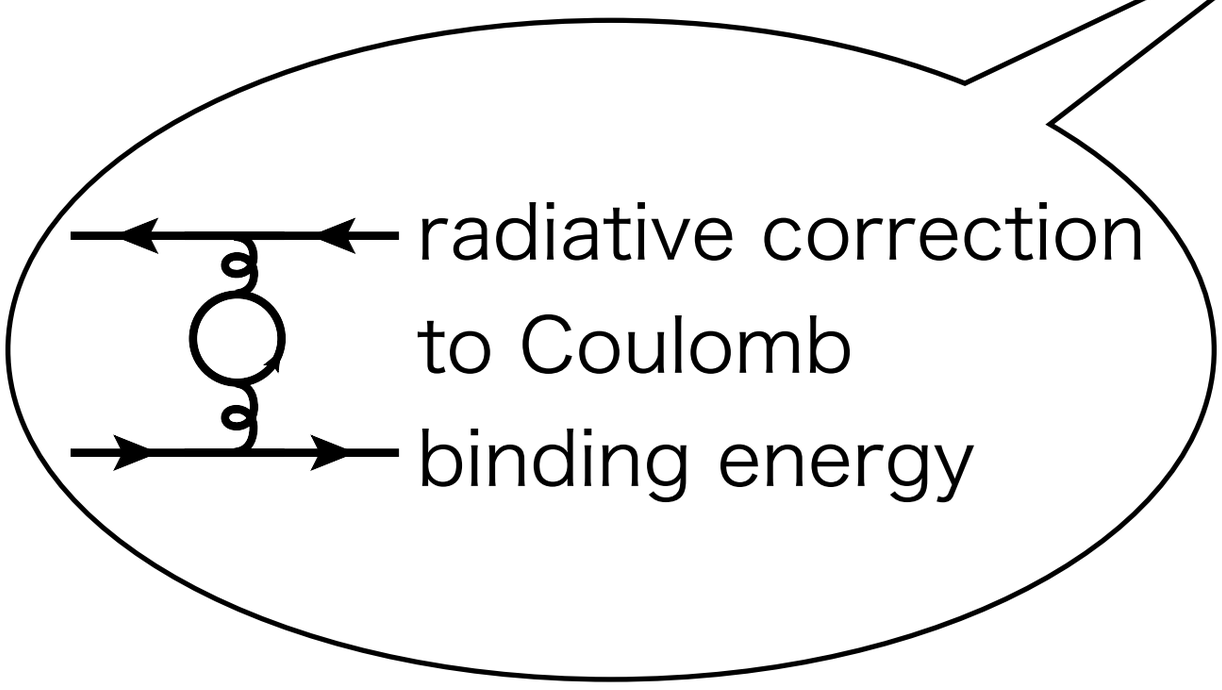

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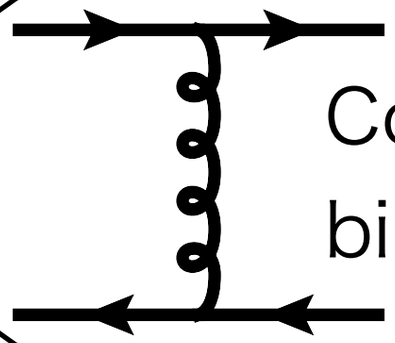
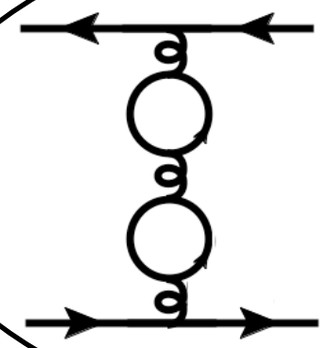
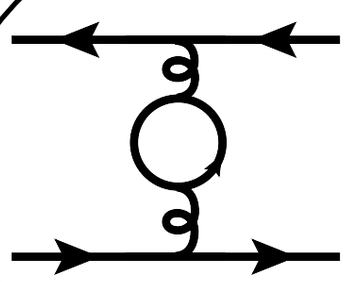


Coulomb
binding energy



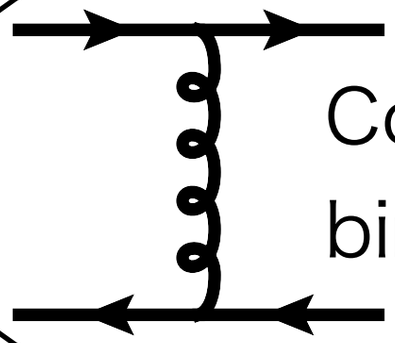
radiative correction
relativistic corrections
(1/m potential)

$$E_{\rm bin} = -\frac{4}{9} m_b^{\rm pole} \alpha_s^2 \left(P_0 + P_1 \frac{\alpha_s}{\pi} + P_2 \frac{\alpha_s^2}{\pi^2} + P_3 \frac{\alpha_s^3}{\pi^3} \right)$$



radiative correction
to Coulomb
binding energy

our current knowledge of binding energy



Coulomb
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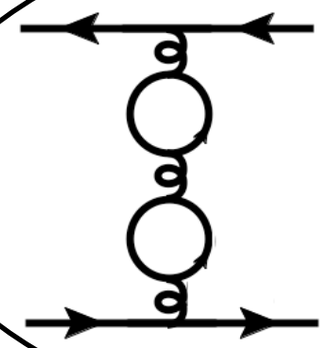

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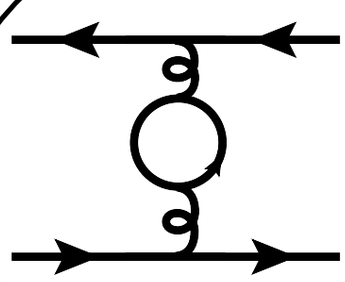

radiative correction
to Coulomb
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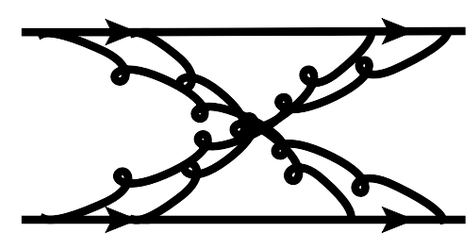
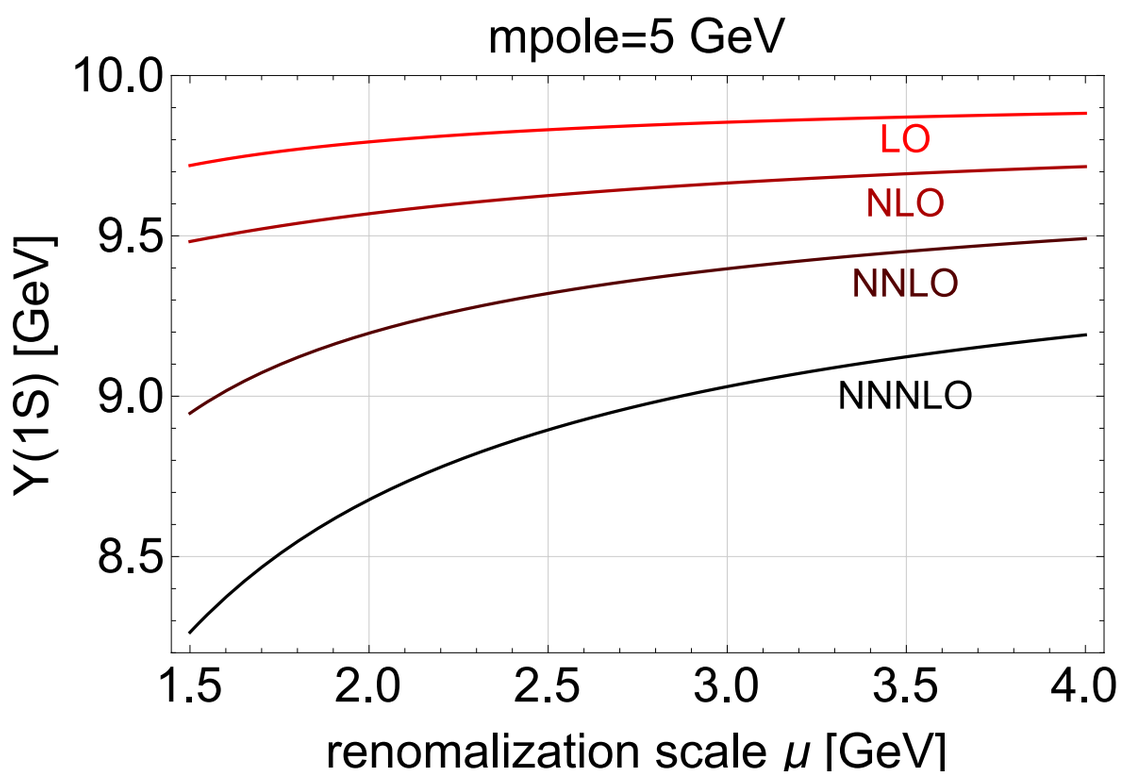


radiative correction
relativistic corrections
(1/m potential)
Ultra-soft correction

Short-distance masses lead good convergence.

$$M_\Upsilon = 2m_b^{\rm pole} + E_{\rm bin}$$

$$E_{\rm bin} = -\frac{4}{9} m_b^{\rm pole} \alpha_s^2 \left(P_0 + P_1 \frac{\alpha_s}{\pi} + P_2 \frac{\alpha_s^2}{\pi^2} + P_3 \frac{\alpha_s^3}{\pi^3} \right)$$



pole mass scheme

Short-distance masses lead good convergence.

$$m_b^{\rm pole} = \overline{m}_b \left(d_0 + d_1 \frac{\alpha_s}{\pi} + d_2 \frac{\alpha_s^2}{\pi^2} + d_3 \frac{\alpha_s^3}{\pi^3} \right)$$

$$M_\Upsilon = 2 m_b^{\rm pole} + E_{\rm bin}$$

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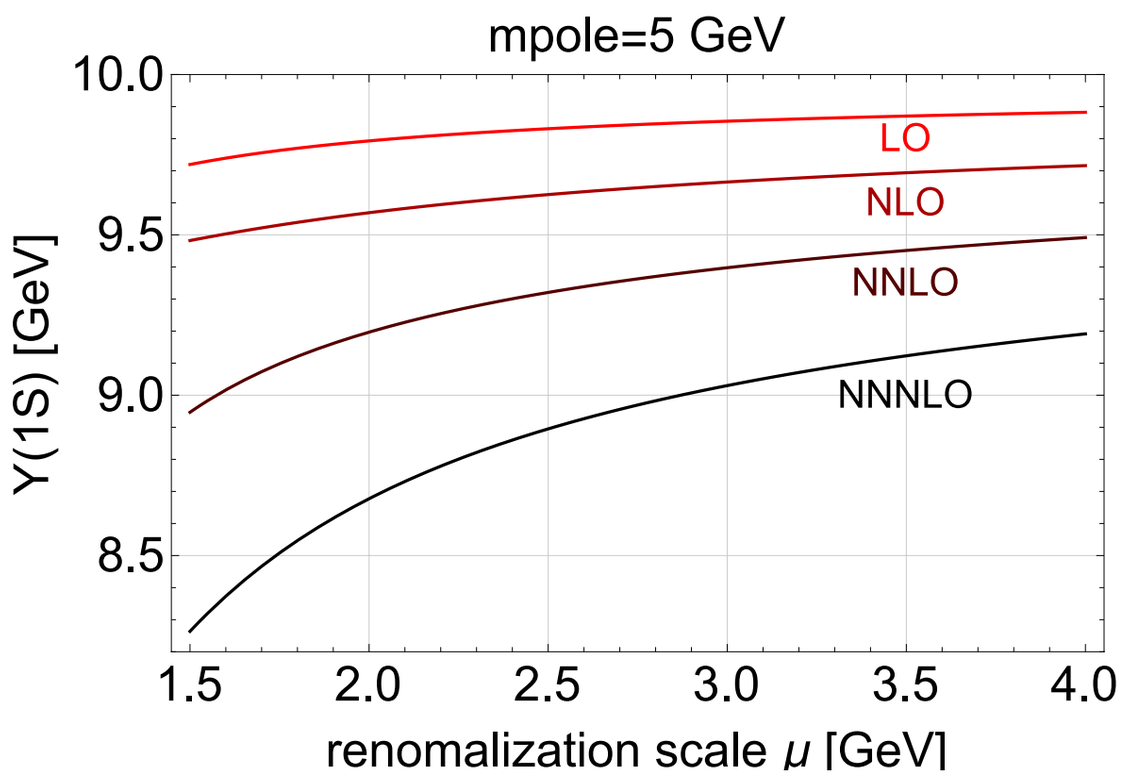
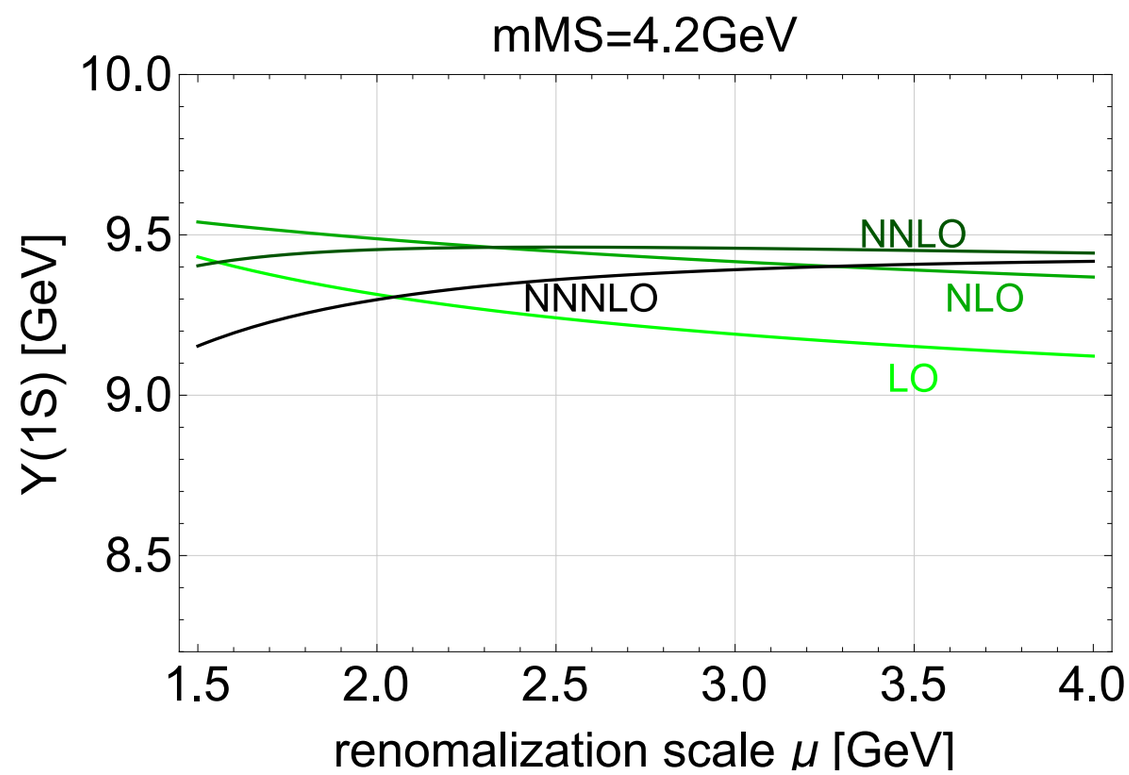


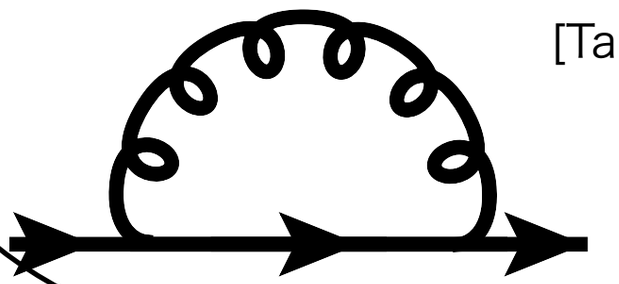
pole mass scheme



$\overline{\rm MS}$ mass scheme

our current knowledge of pole-$\overline{\text{MS}}$ mass relation

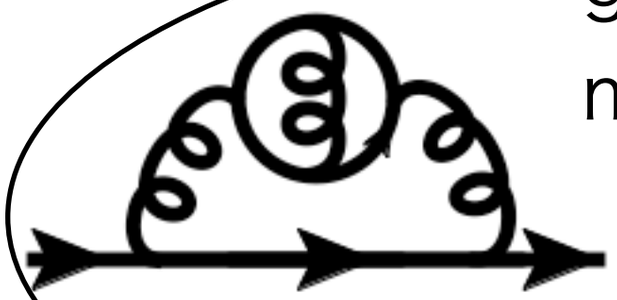
gluon correction only



[Tarrach, '81]

gluon

massless quark



[Chetyrkin, Steinhauser, '99]

[Melnikov, Ritbergen '99]

massive quark

[Bekavac, Grozin, Seidel, Steinhauser, '07]

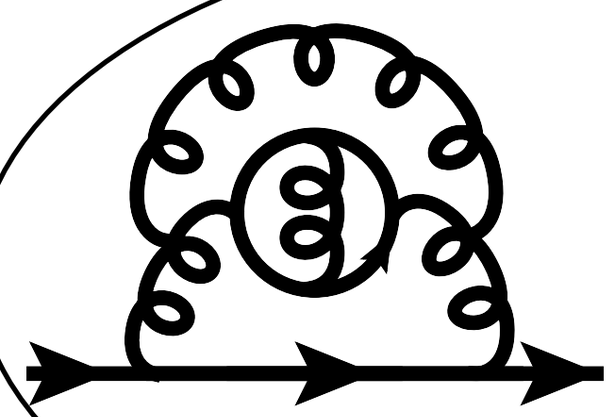
$$m_b^{\text{pole}} = \overline{m}_b \left(d_0 + d_1 \frac{\alpha_s}{\pi} + d_2 \frac{\alpha_s^2}{\pi^2} + d_3 \frac{\alpha_s^3}{\pi^3} \right) + \mathcal{O}(\alpha_s^4)$$

gluon

massless quark

massive quark

[Gray, Broadhurst, Grafe, Schliche, '90]

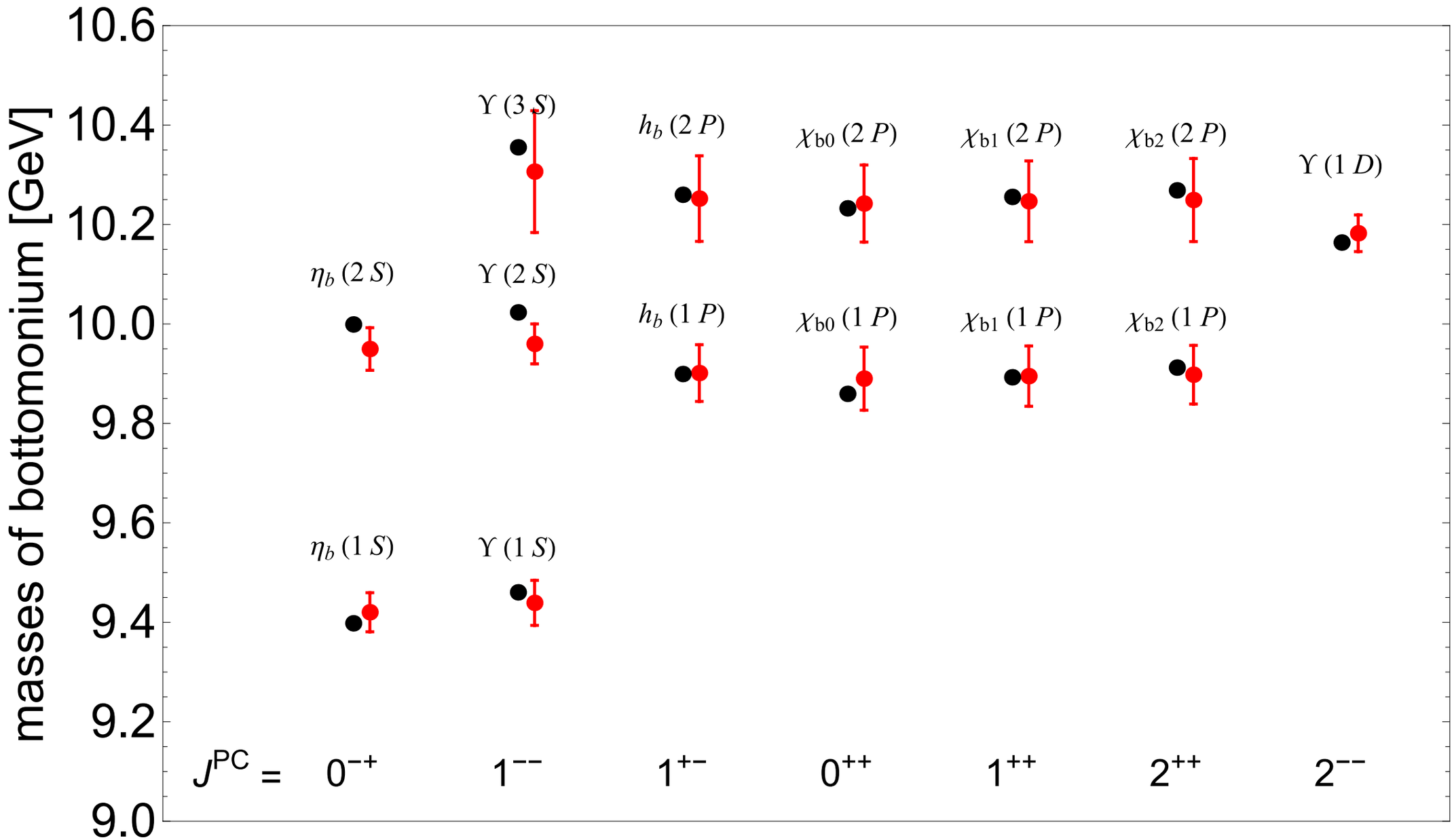


gluon

massless quark

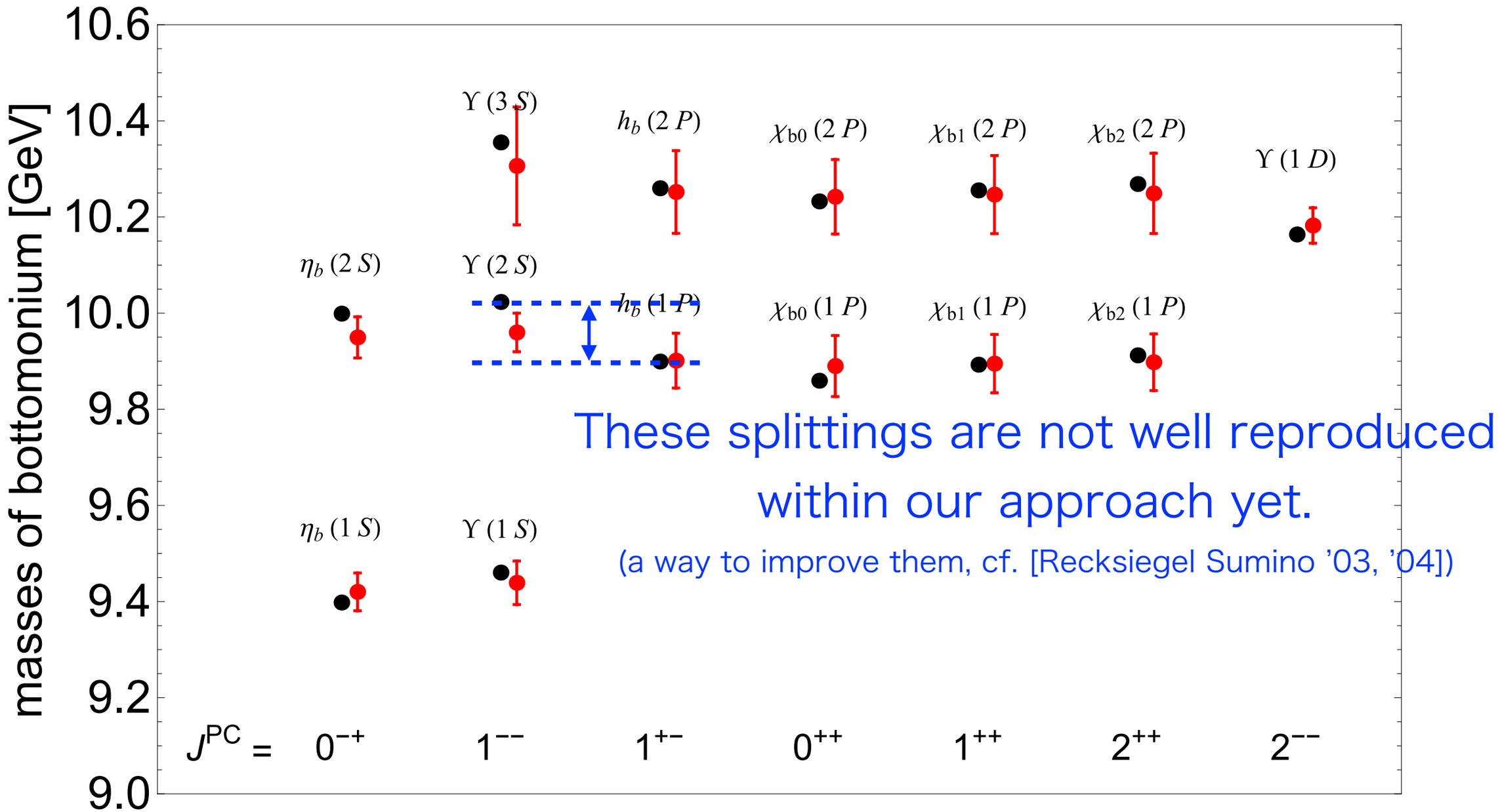
[Marquard, Smirnov, Smirnov, Steinhauser, '15]

bottomonium spectrum in the perturbative QCD at NNNLO



private calculation based on [Kiyo, Sumino '14]
The finite-charm-mass correction is not included.
Theoretical uncertainty is estimated by NNLO-NNNLO.

bottomonium spectrum in the perturbative QCD at NNNLO



These splittings are not well reproduced within our approach yet.

(a way to improve them, cf. [Recksiegel Sumino '03, '04])

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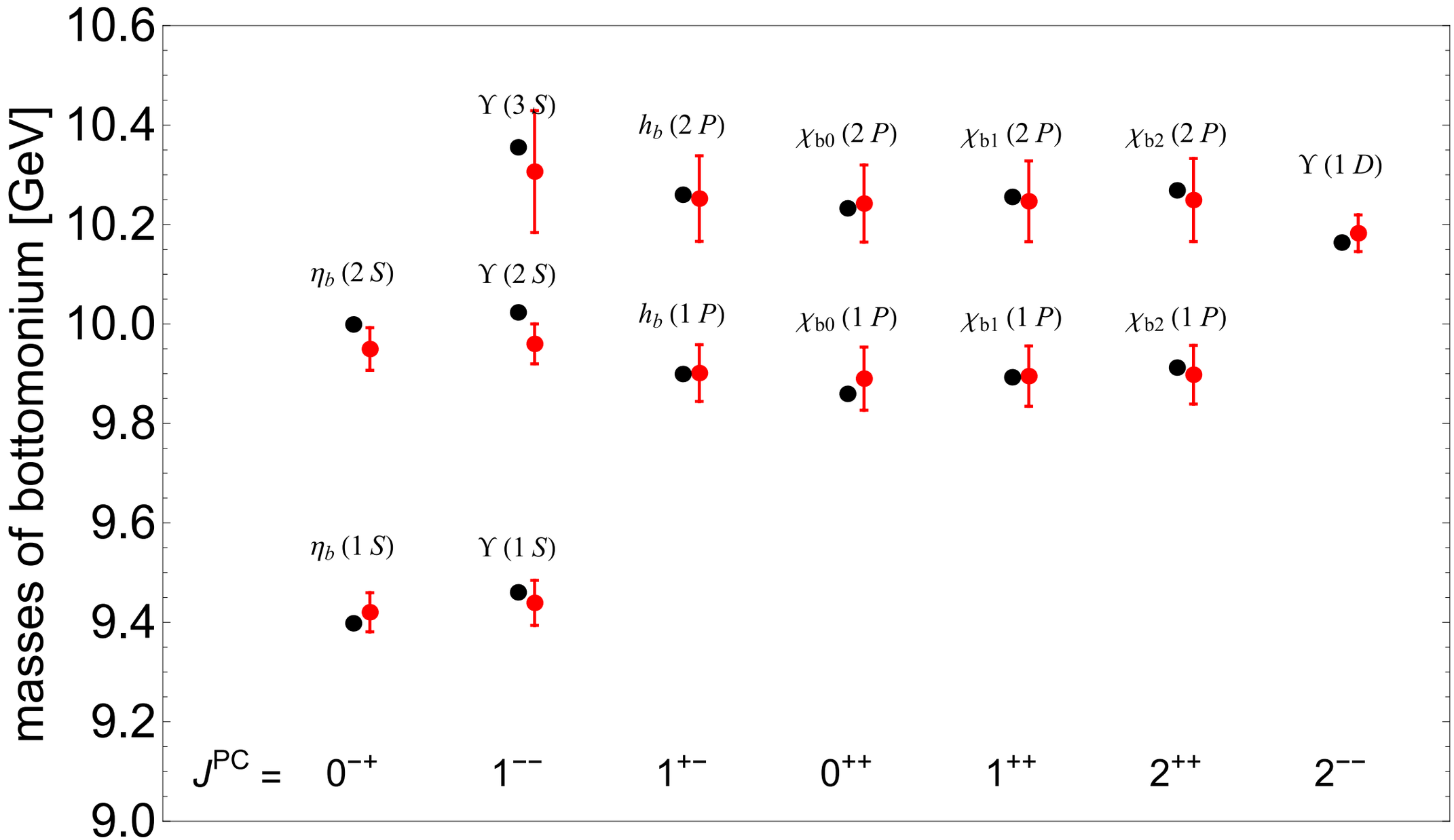

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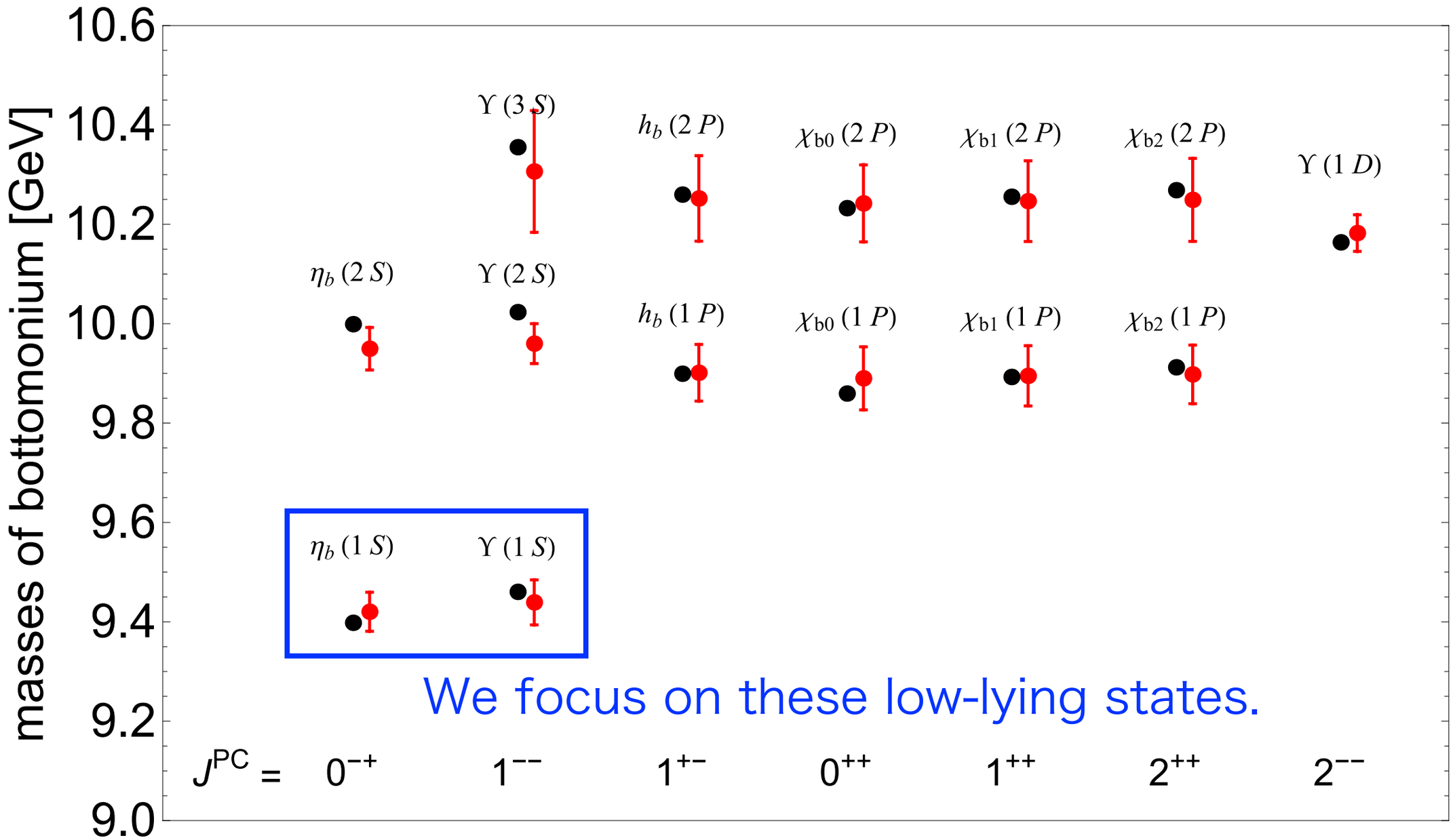

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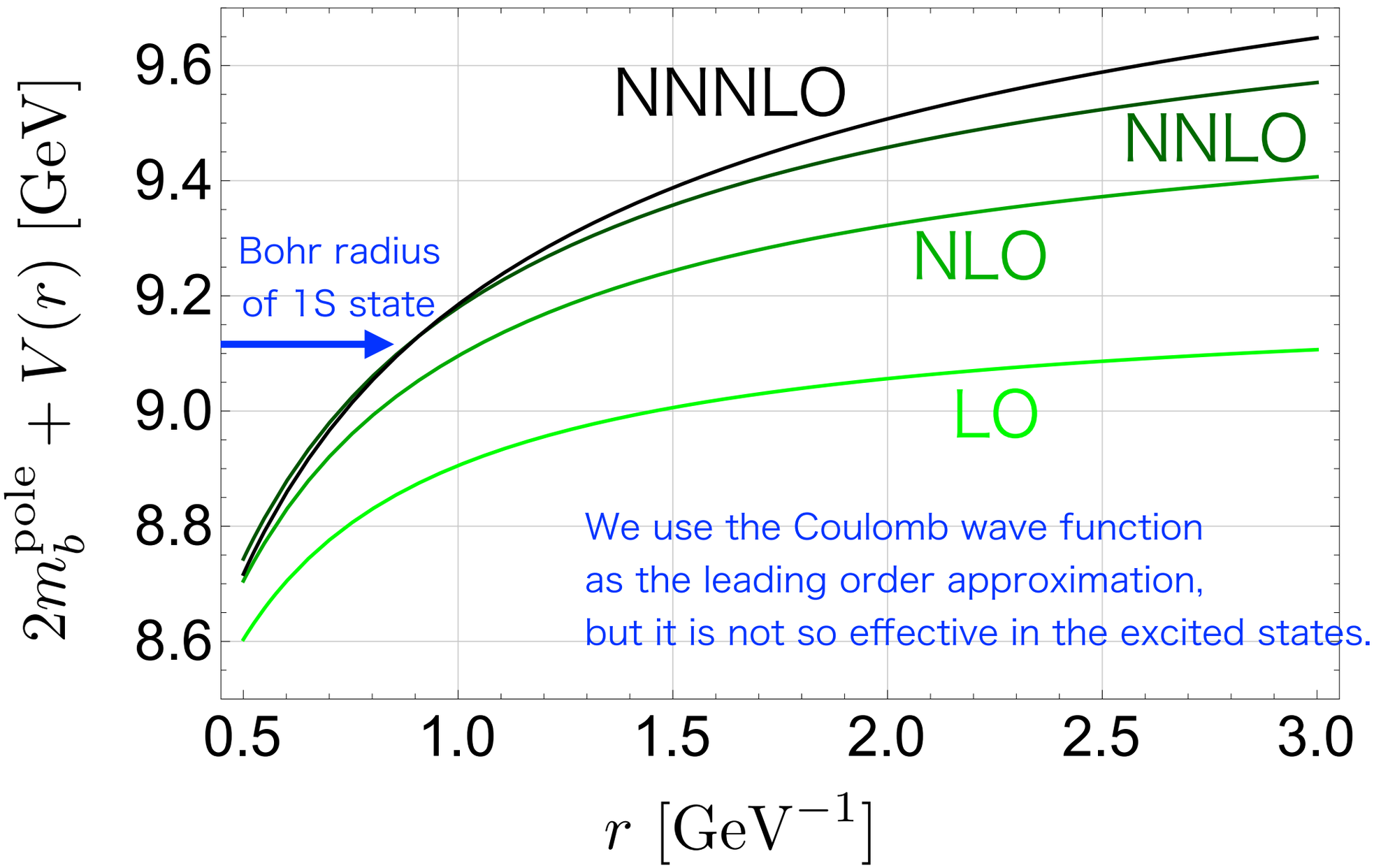
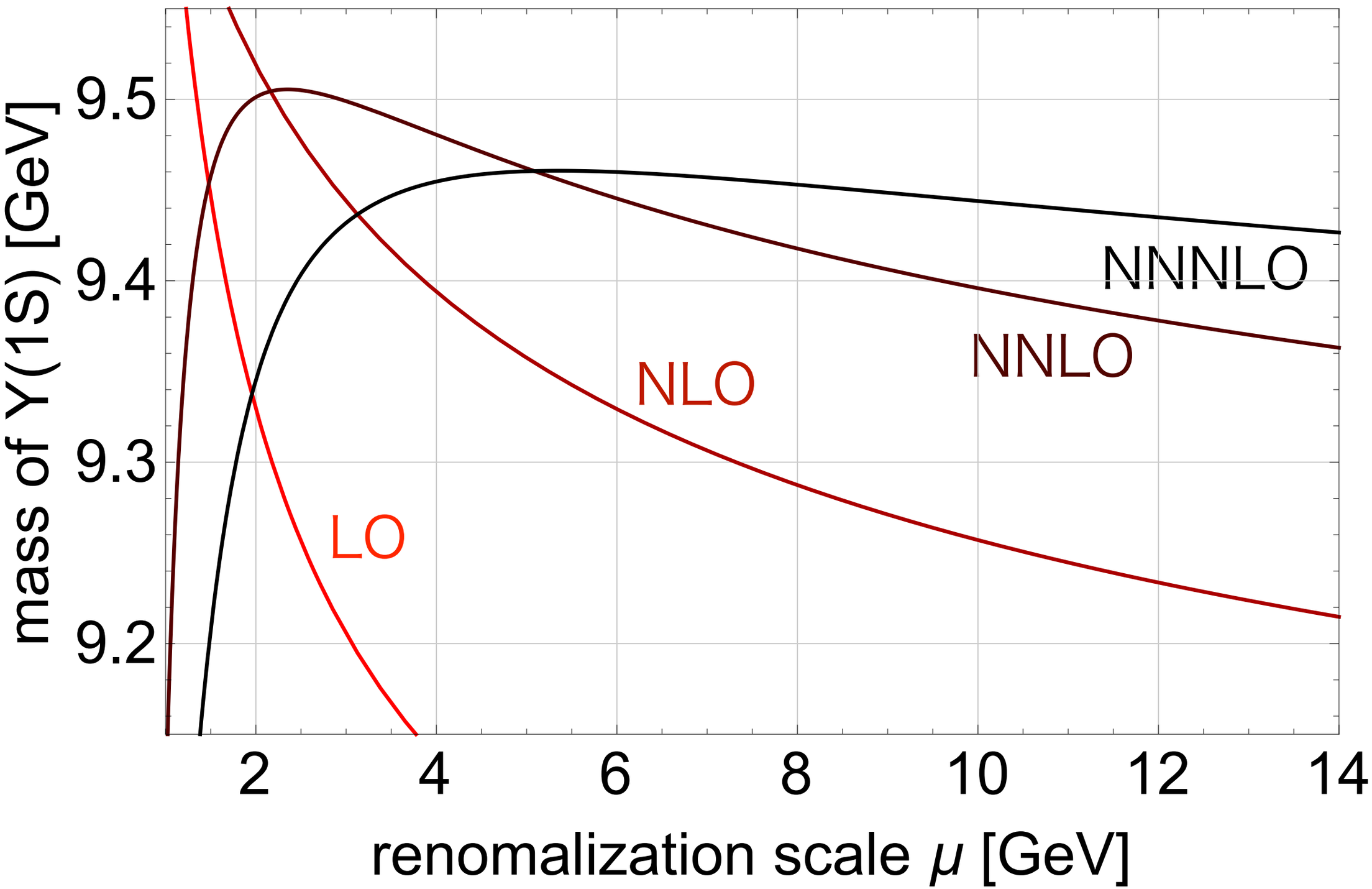
Theoretical uncertainty is estimated by NNLO-NNNLO.

We focus on the lowest-lying states ($\Upsilon(1S), \eta_b(1S)$)
because the excited states are
more sensitive to higher order corrections.



Convergence and scale independence become better as we go higher order.



Estimation of higher order corrections by the scale dependence

$$M_\Upsilon = 2\overline{m}_b \underbrace{\left[1 + \overbrace{f_1\alpha_s + f_2\alpha_s^2 + f_3\alpha_s^3 + f_4\alpha_s^4}^{\text{renormalization scale dependent}} + \overbrace{\cdots}^{\text{renormalization scale dependent}}\right]}$$

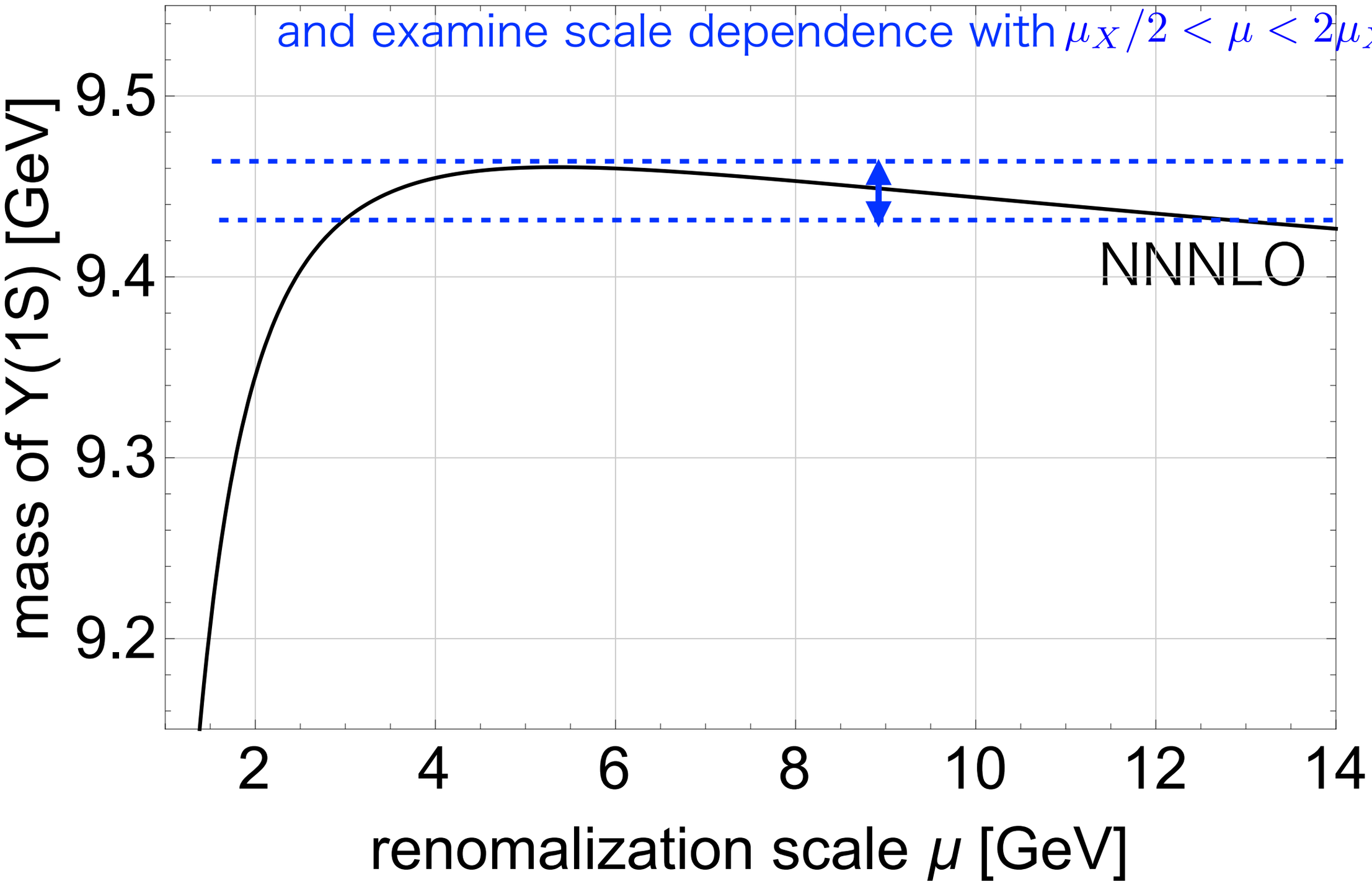
Formally, scale dependence disappears when all order corrections are included.

$\alpha_s(\mu)$: $\overline{\text{MS}}$ renormalized strong coupling constant

$f_i(\mu)$: scale-dependent coefficients

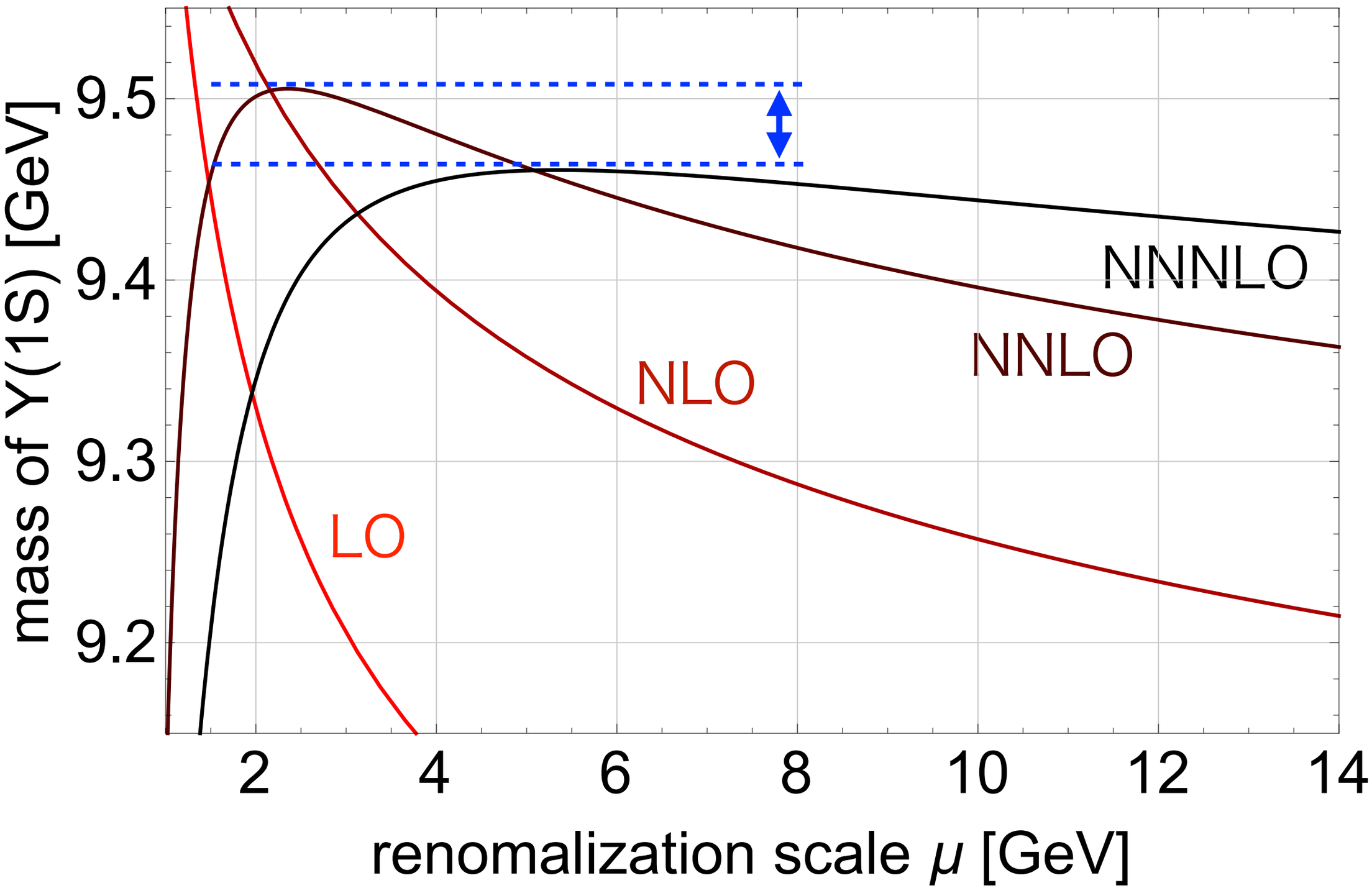
Uncertainty estimation by scale variation

We adopt the value at minimal sensitivity scale μ_X,
and examine scale dependence with $\mu_X/2 < \mu < 2\mu_X$.

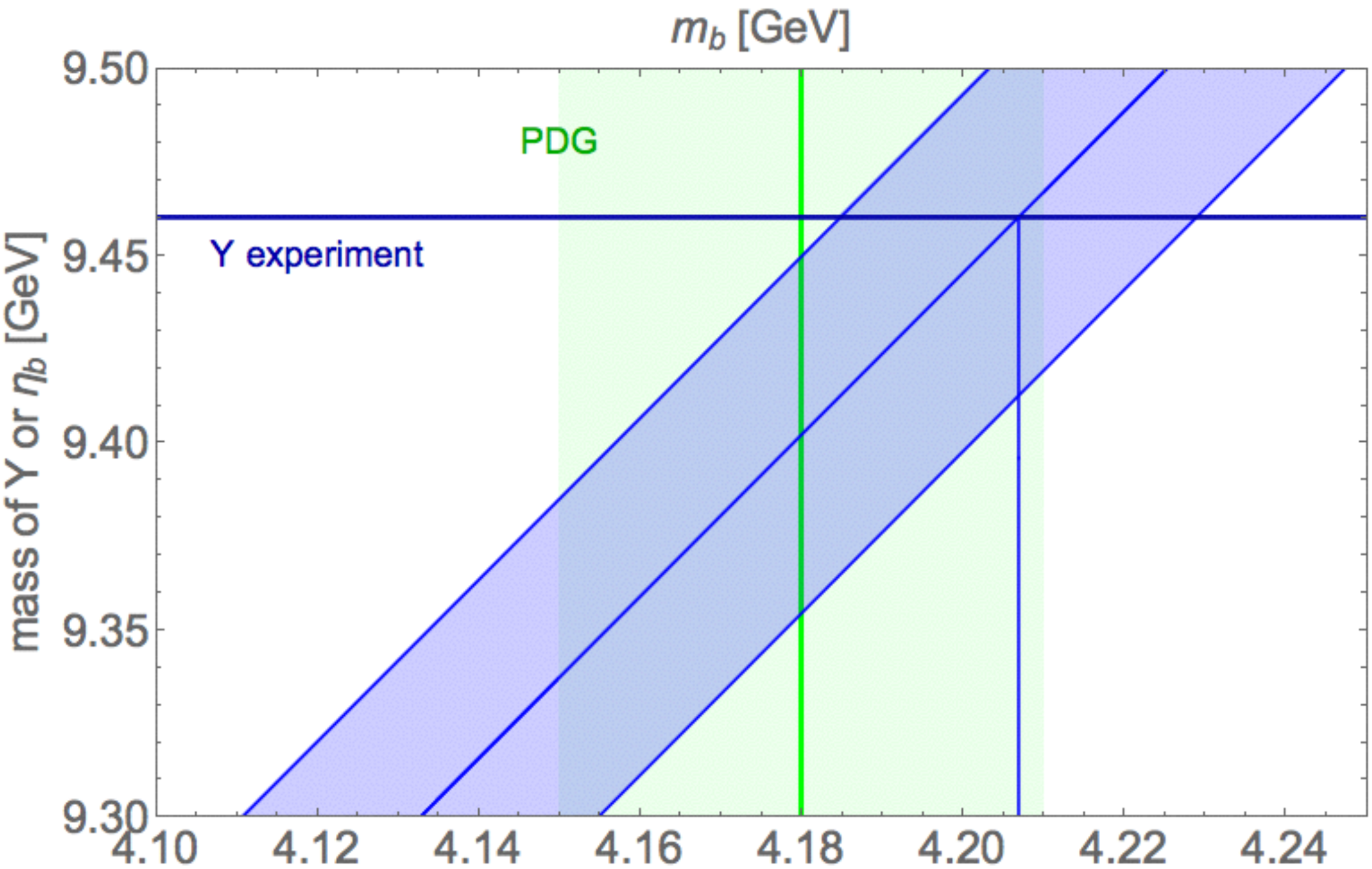


Uncertainty estimation by NNLO-NNNLO

We adopt the difference between NNLO and NNNLO.



result (bottom mass)



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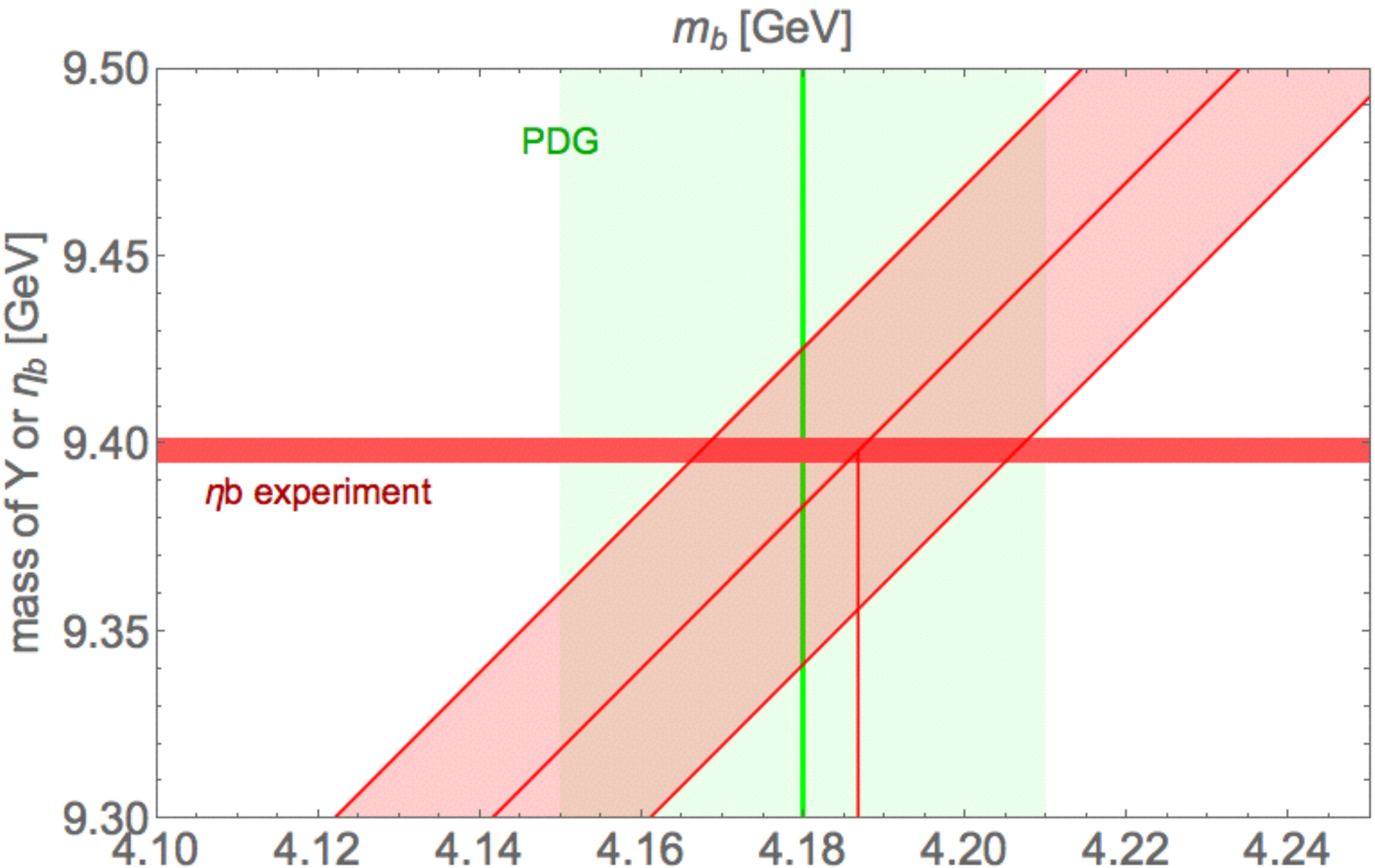

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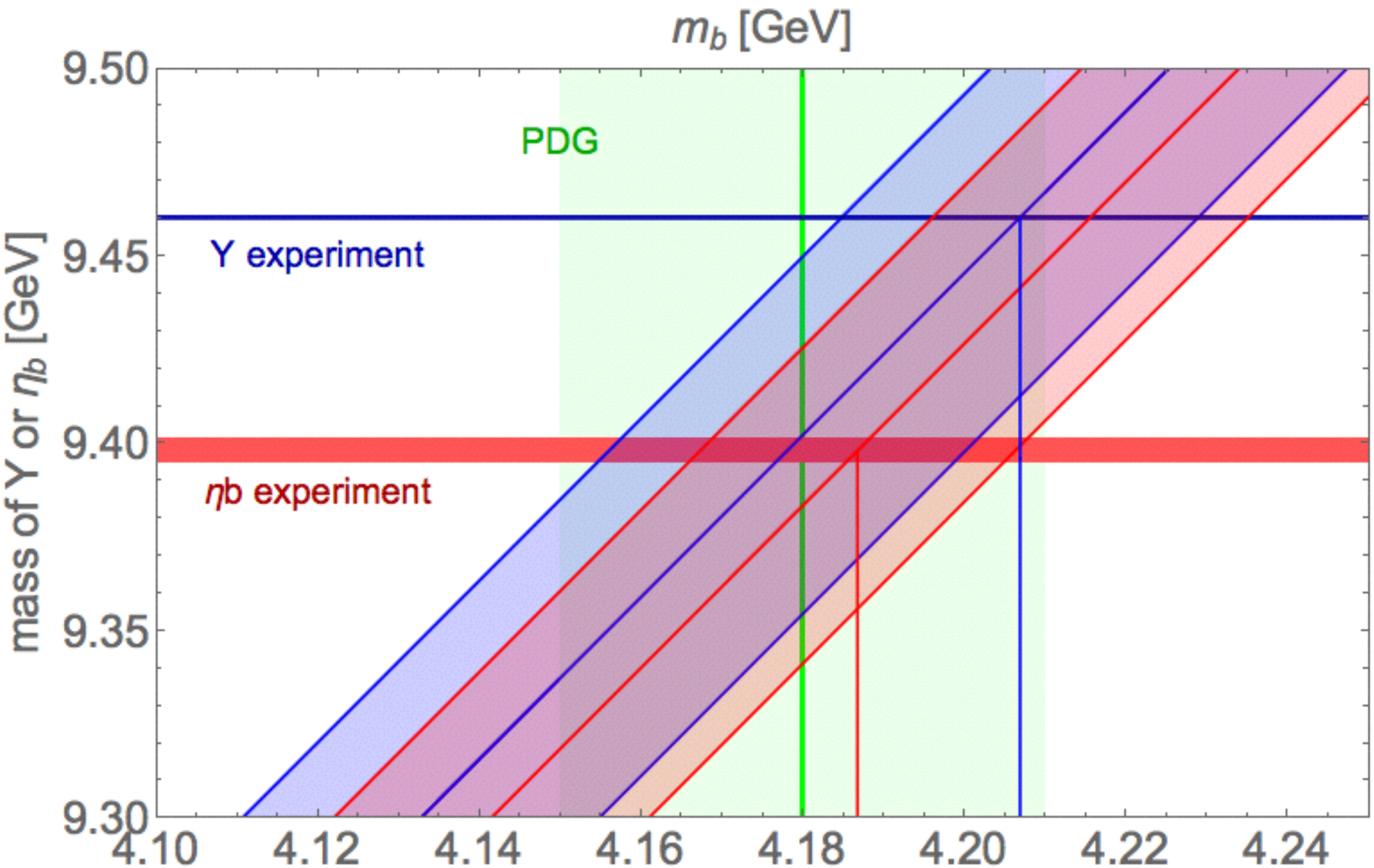
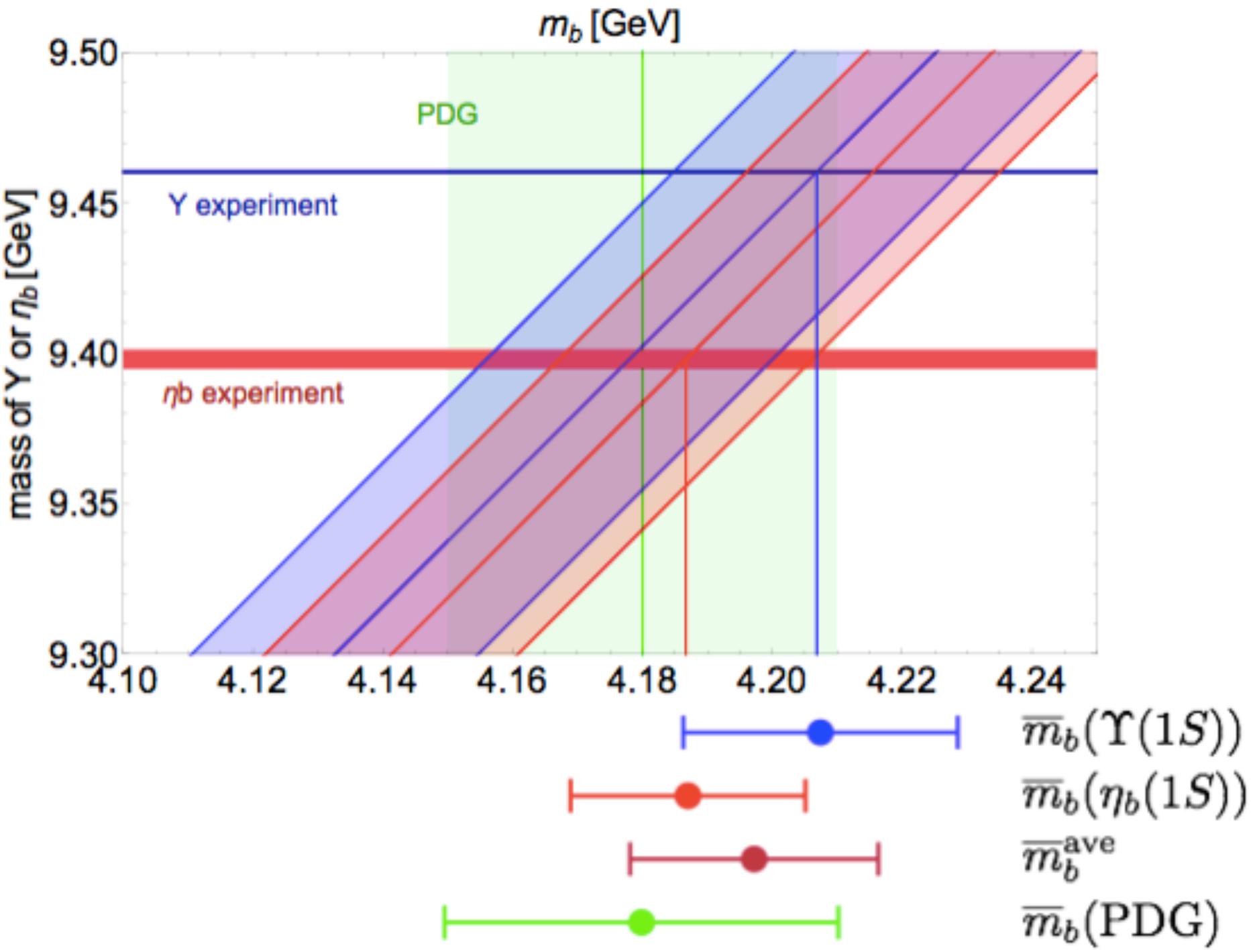


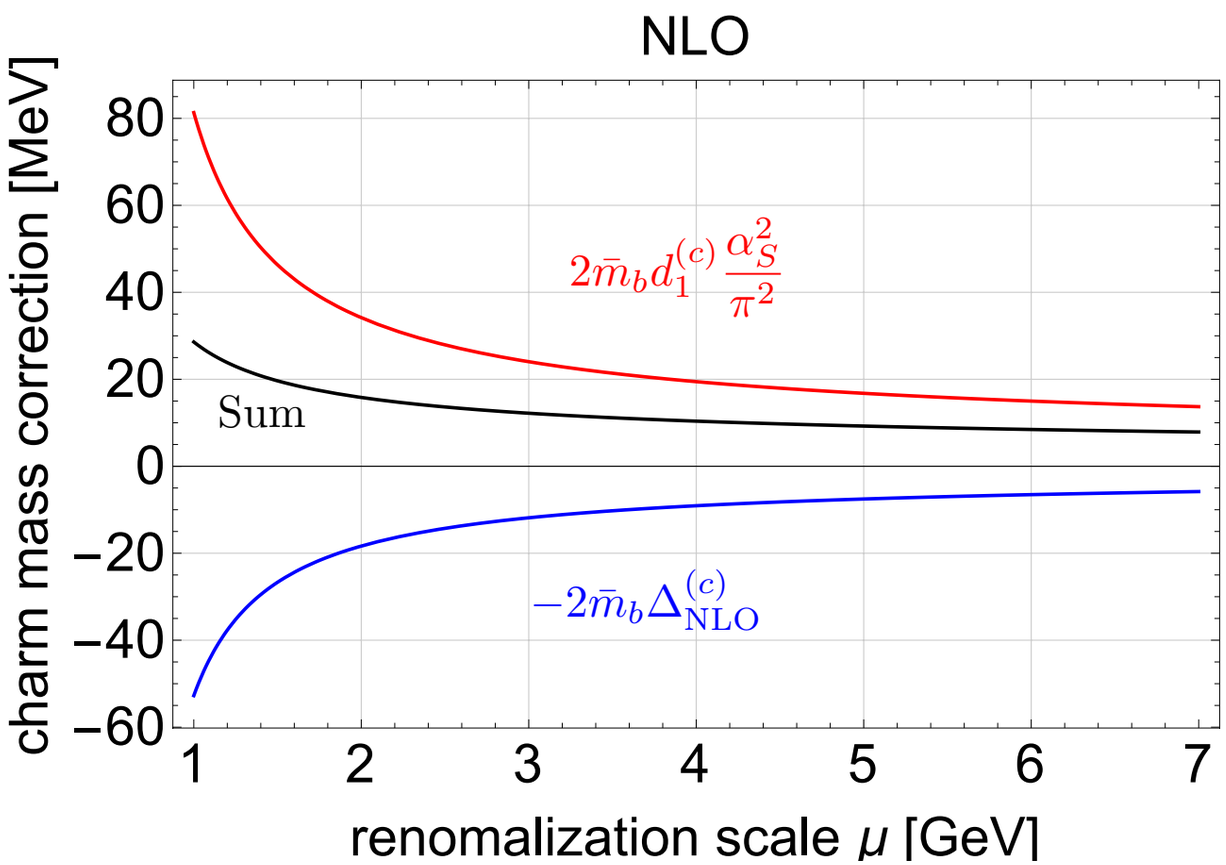
result (bottom mass)

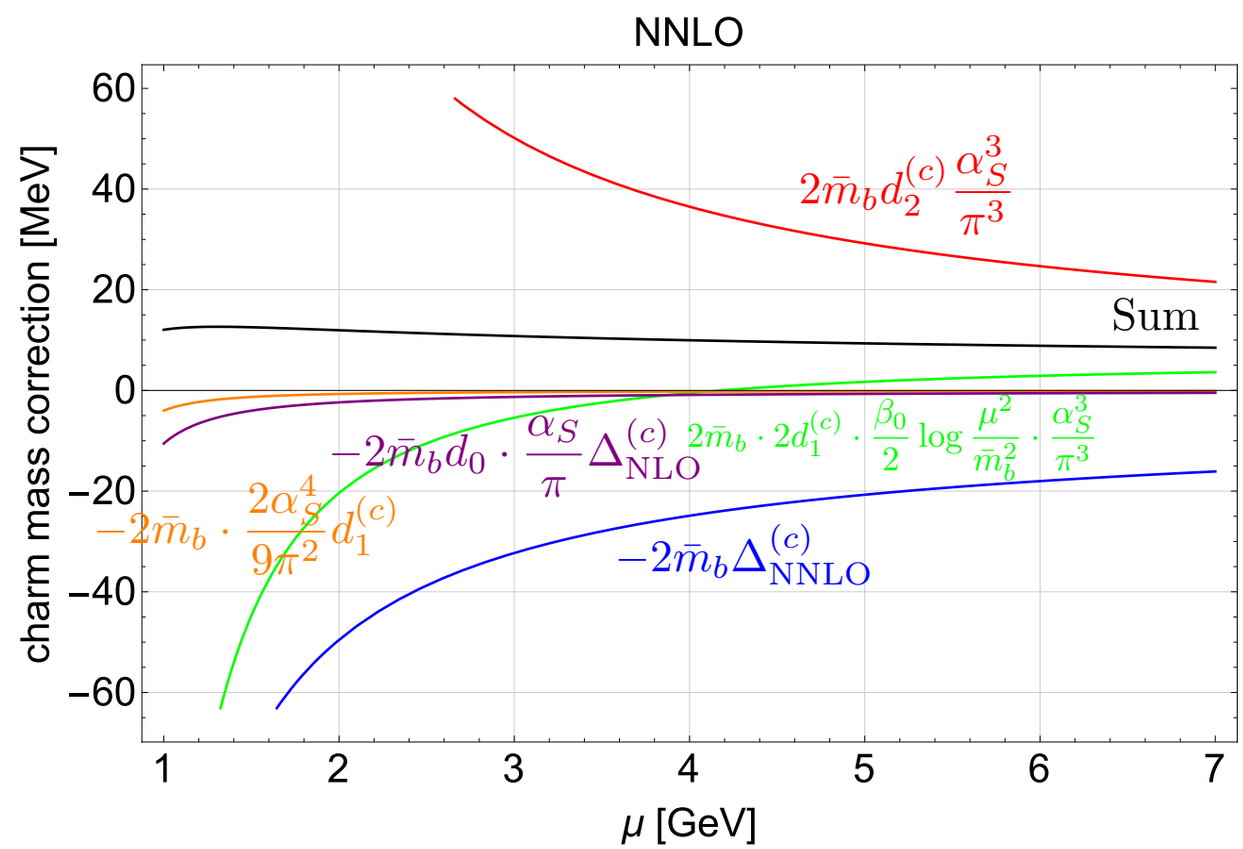
$$\overline{m}_b^{\rm ave} = 4197 \pm 2\ (d_3)\ \ \pm 6\ (\alpha_s)\ \ \pm 20\ ({\rm h.o.}) \pm 5\ (m_c)\quad {\rm MeV}$$



finite-charm-mass correction

We adopt the scheme in which
most of the bottomonium mass is written
in the four massless quark theory
and the finite-charm-mass correction is added to them.

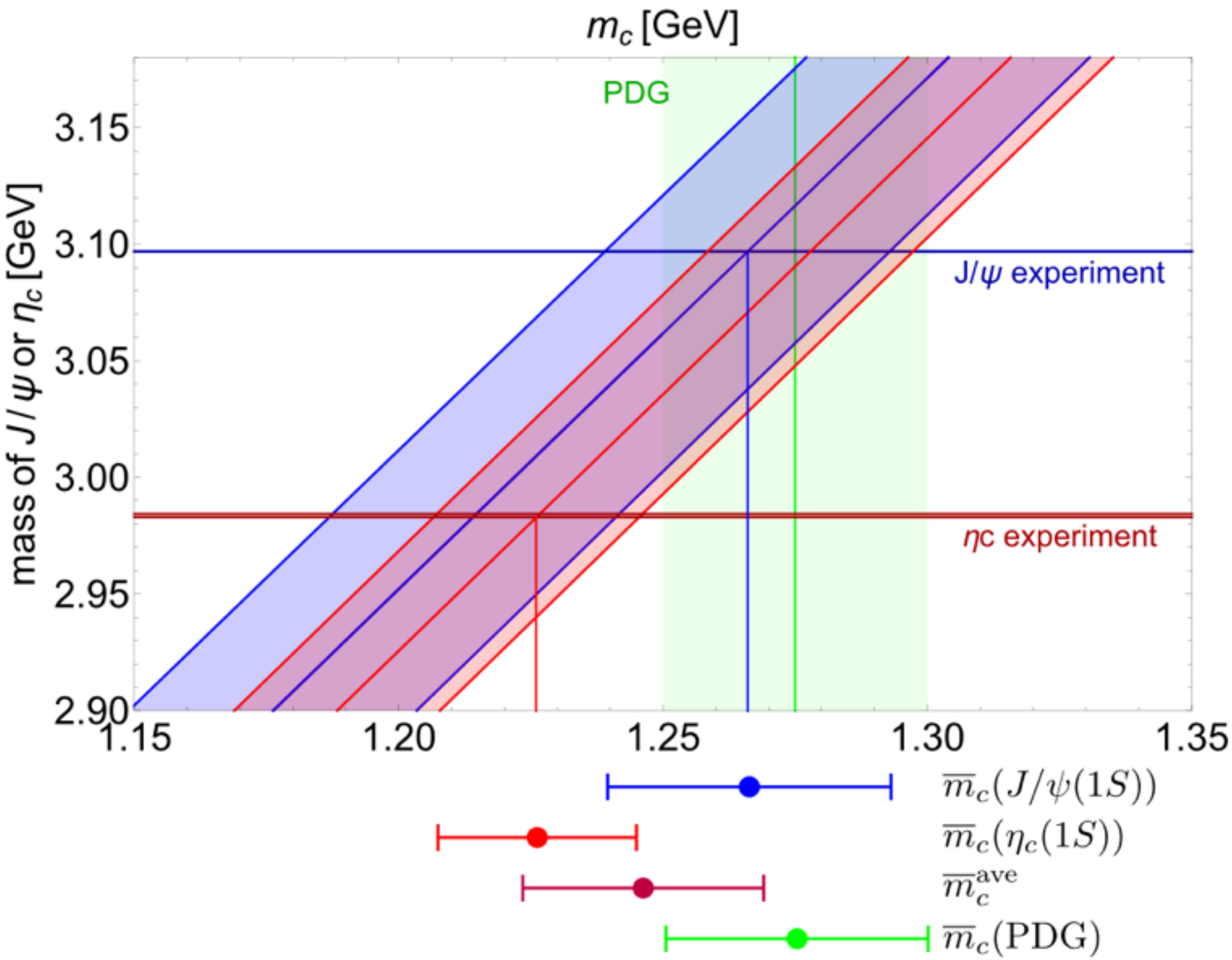




Cancellation between the quark self energy correction
and binding energy correction is observed.

result (charm mass)

$$\overline{m}_c^{\rm ave} = 1246 \pm 2\ (d_3)\ \ \pm 4\ (\alpha_s)\ \ \pm 23\ ({\rm h.o.})\quad {\rm MeV}$$

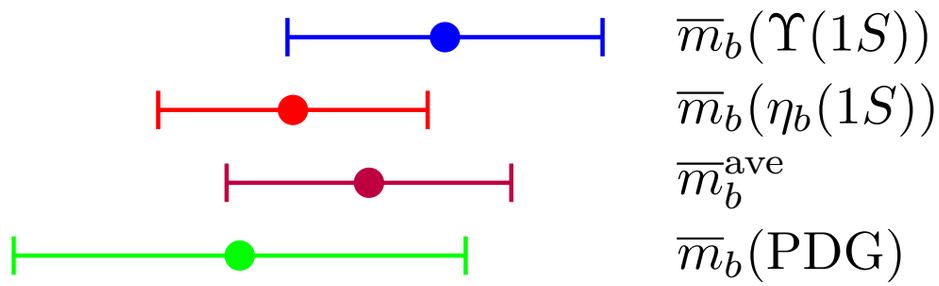


Summary

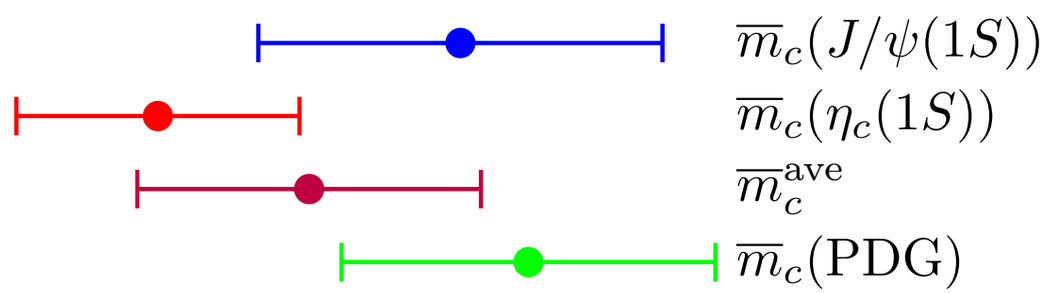
- We determined m_c and m_b from charmonium and bottomonium 1S states in the NNNLO perturbative QCD.

- The theoretical uncertainty is estimated in a systematic way.

$$\overline{m}_b^{\rm ave} = 4197 \pm 2\ (d_3)\ \pm 6\ (\alpha_s)\ \pm 20\ ({\rm h.o.}) \pm 5\ (m_c) \quad {\rm MeV}$$

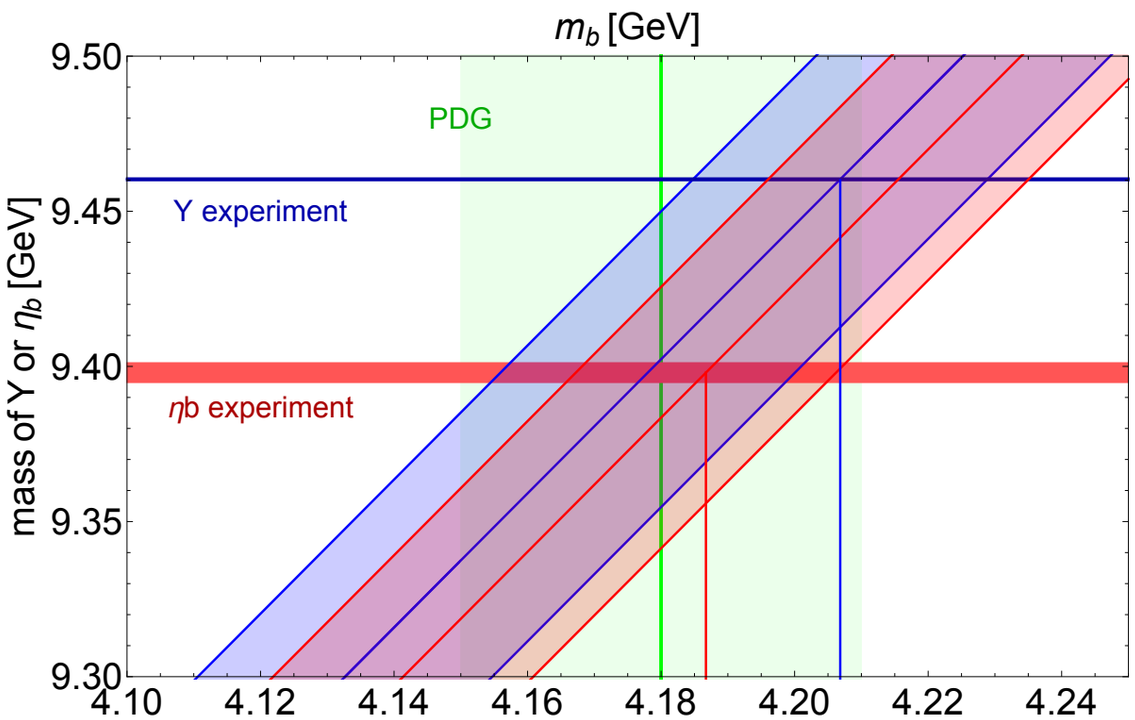


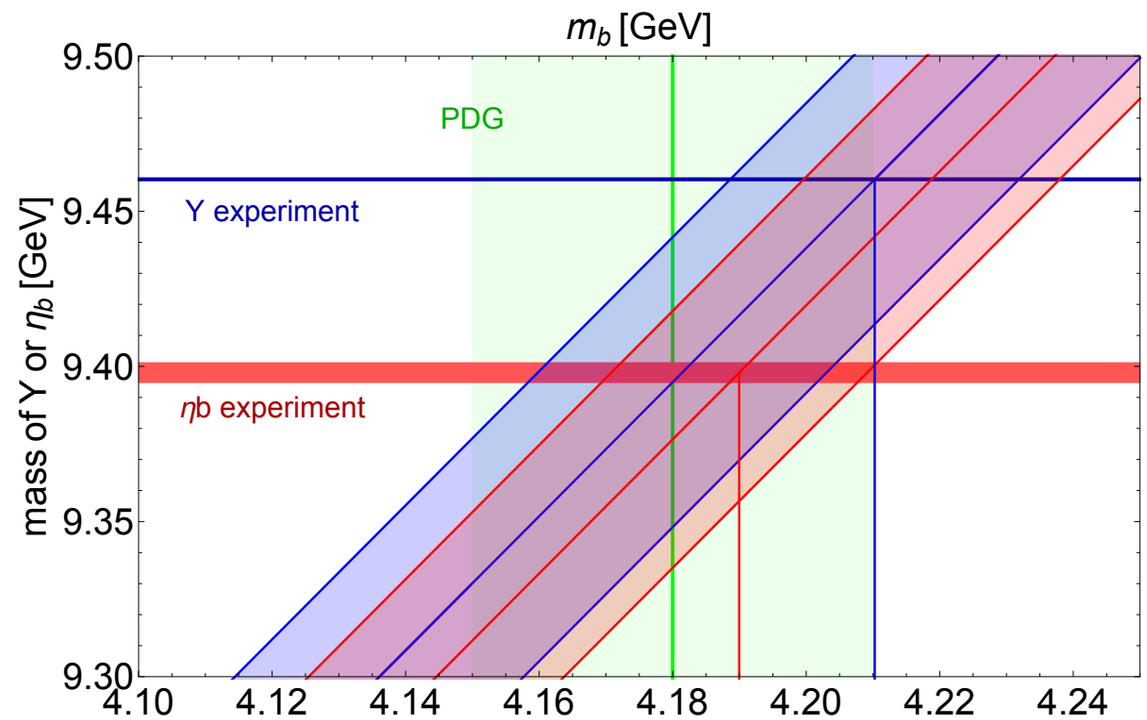
$$\overline{m}_c^{\rm ave} = 1246 \pm 2\ (d_3)\ \pm 4\ (\alpha_s)\ \pm 23\ ({\rm h.o.}) \quad {\rm MeV}$$



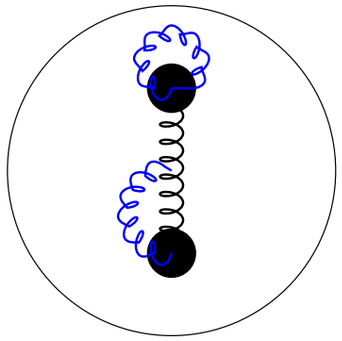
Updated value

$$\alpha_s(m_Z) = 0.1185 \pm 0.0006 \rightarrow 0.1181 \pm 0.0013$$

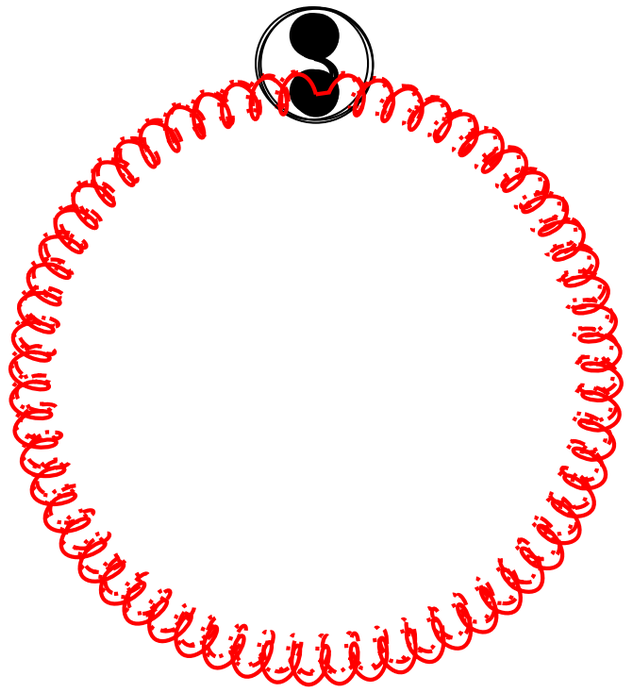




Why infrared effects cancel? (intuitive view)



Bottomonium bound state receives UV (short-distance) effects.



Bottomonium is color singlet state, so gluons of long wave length decouple from the bottomonium. Thus IR (long-distance) effects on the bottomonium decreases.