Long-range properties of 1S bottomonium states

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QWG2016

June 6 - 10, 2016
Pacific Northwest National Laboratory
Richland, WA, USA

Work in collaboration with

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— Jaume Tarrús-Castellà

— Antonio Vairo

Motivation

Charmonium binding in nuclei

- an exotic nuclear state

Brodsky, Schmidt & de Téramond, PRL 64, 1011 (1990)

— Nucleons and charmonium have no valence quarks in common

— Interaction has to proceed via gluons – QCD van der Waals

— No Pauli Principle – no short-range repulsion

— Also, binding via D,D* meson loop - interaction with nucleons

BE ~ 10 - 20 MeV

GK, A. W. Thomas & K. Tsushima PLB 697, 136 (2011)
K. Tsushima, D. Lu, GK & A. W. Thomas PRC 83, 065208 (2011)

J/Ψ binding to nuclei

Generically: two independent mechanisms

— van der Waals force - octet intermediate state, large N_c

M.E. Luke, A.V. Manohar & M.J. Savage PLB 288, 355(1992)

— D,D* meson-loop – color singlet intermediate state

D mesons interact with the nuclear mean fields

S. H. Lee & C.M. Ko PRC 67, 038202 (2003)
GK, A. W. Thomas & K. Tsushima PLB 697, 136 (2011)
K. Tsushima, D.H. Lu, GK & A.W. Thomas PRC 83, 065208 (2011)

Bhanot-Peskin theory

— large N_c

J/Ψ as a small color electric dipole

$$\Delta m_\psi(\rho_B) = -\frac{1}{9}\int dk^2 \left|\frac{\partial\psi(k)}{\partial k}\right|^2 \frac{k}{k^2/m_c+\epsilon}\langle\alpha_s E^2/\pi\rangle_N \frac{\rho_B}{2m_N}$$

$\psi(k)$: charmonium wave function

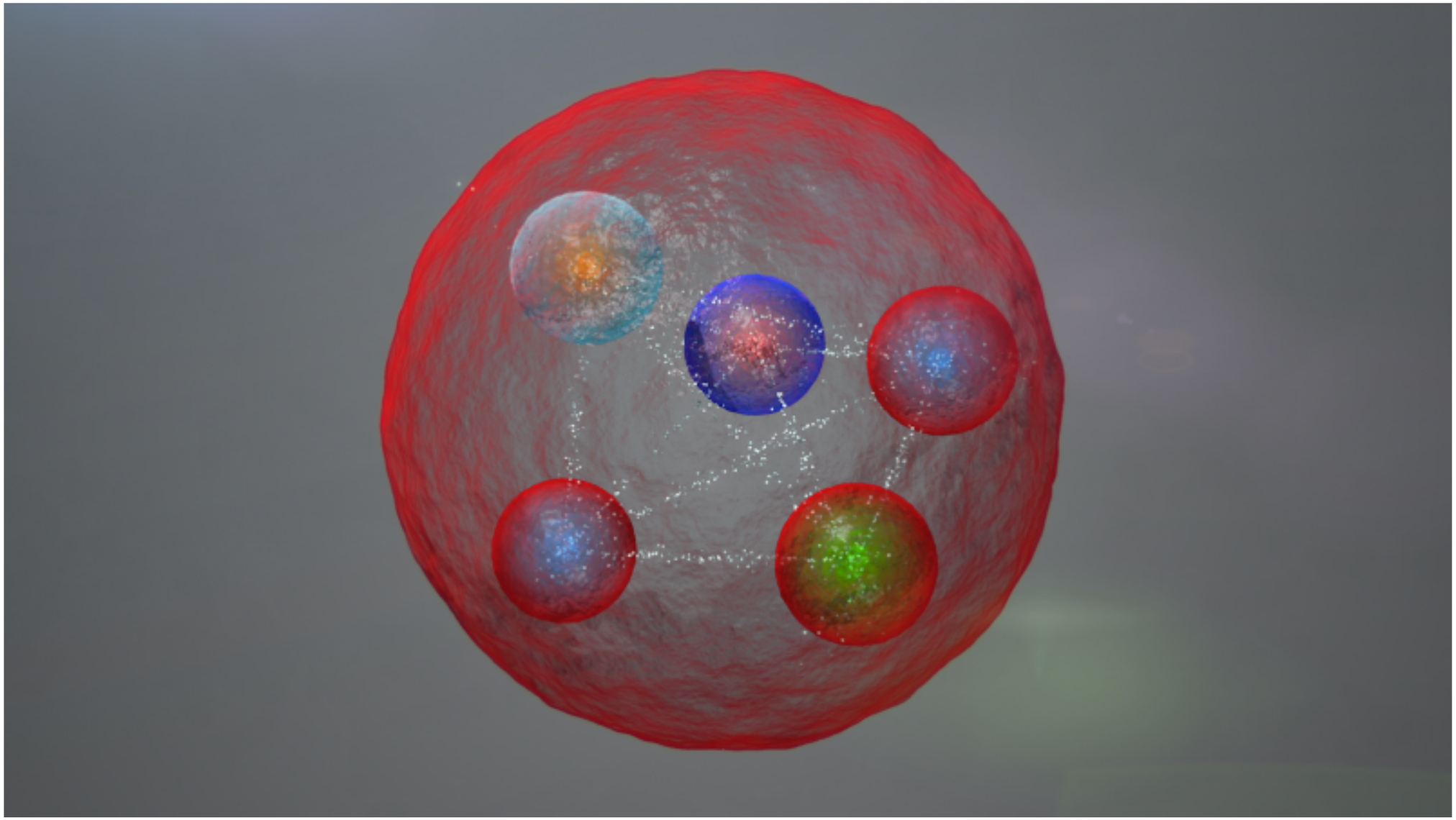
ρ_B : nuclear density

m_c, m_N : masses charm quark and nucleon

$\epsilon = 2m_c - m_\psi$: energy shift octet – charmonium

Note: intermediate state is noninteracting

Intact J/Ψ "inside" the proton



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Numerical results

$$\langle \alpha_s E^2/\pi \rangle_N = 0.5\,\mathrm{GeV}^2$$

ψ : Gaussian

$\langle r^2 \rangle$: from Cornell potential model

$$\Delta m_\psi = -8\,\mathrm{MeV}$$

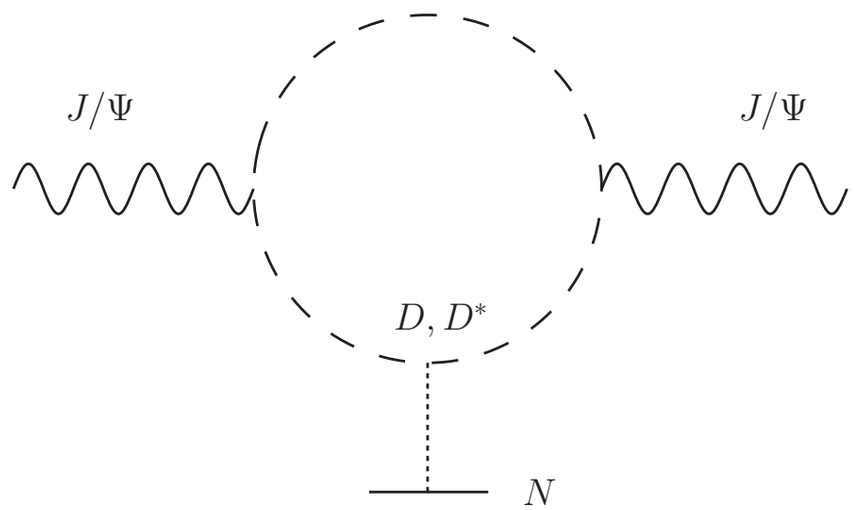
at normal nuclear matter density

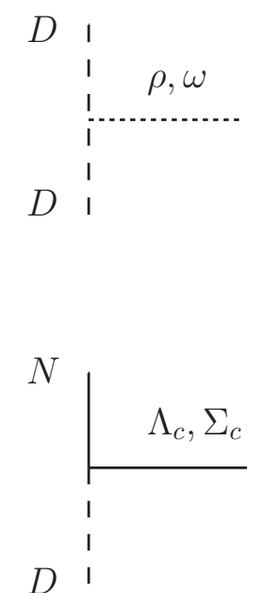
Sibirtsev & Voloshin, PRD 71, 076005 (2005)

$\mathrm{J/\Psi\,N}$ cross section > 17 mb

$$\Delta m_\psi = -21\,\mathrm{MeV}$$

D,D*-meson loops





Calculate loop with effective Lagrangians

– need coupling constants & form factors

– need medium dependence of D masses

GK, A. W. Thomas & K. Tsushima PLB 697, 136 (2011)
K. Tsushima, D. Lu, GK & A. W. Thomas PRC 83, 065208 (2011)

J/Ψ single-particle energies in nuclei

— from Klein-Gordon equation

		$\Lambda_{D,D^*} = 1500$ MeV	$\Lambda_{D,D^*} = 2000$ MeV
		E (MeV)	E (MeV)
${}^{4}_{\Psi}$He	$1s$	−4.19	−5.74
${}^{12}_{\Psi}$C	$1s$	−9.33	−11.21
	$1p$	−2.58	−3.94
${}^{16}_{\Psi}$O	$1s$	−11.23	−13.26
	$1p$	−5.11	−6.81
${}^{40}_{\Psi}$Ca	$1s$	−14.96	−17.24
	$1p$	−10.81	−12.92
	$1d$	−6.29	−8.21
	$2s$	−5.63	−7.48
${}^{90}_{\Psi}$Zr	$1s$	−16.38	−18.69
	$1p$	−13.84	−16.07
	$1d$	−10.92	−13.06
	$2s$	−10.11	−12.22
${}^{208}_{\Psi}$Pb	$1s$	−16.83	−19.10
	$1p$	−15.36	−17.59
	$1d$	−13.61	−15.81
	$2s$	−13.07	−15.26

K. Tsushima, D. Lu, GK & A. W. Thomas PRC 83, 065208 (2011)

PHYSICAL REVIEW D **91**, 114503 (2015)

Quarkonium-nucleus bound states from lattice QCD

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(Received 9 November 2014; published 11 June 2015)

Quarkonium-nucleus systems are composed of two interacting hadronic states without common valence quarks, which interact primarily through multigluon exchanges, realizing a color van der Waals force. We present lattice QCD calculations of the interactions of strange and charm quarkonia with light nuclei. Both the strangeonium-nucleus and charmonium-nucleus systems are found to be relatively deeply bound when the masses of the three light quarks are set equal to that of the physical strange quark. Extrapolation of these results to the physical light-quark masses suggests that the binding energy of charmonium to nuclear matter is $B^{\mathrm{NM}}_{\mathrm{phys}} \lesssim 40$ MeV.

DOI: 10.1103/PhysRevD.91.114503 PACS numbers: 11.15.Ha, 12.38.Gc, 13.40.Gp

Models

TABLE I. Estimates for the binding energies of charmonium to light nuclei and nuclear matter (in MeV) from selected models. A "$*$" indicates the system is predicted to be unbound, while entries with center dots indicate that the system was not addressed.

	Binding energy (MeV)			Binding energy (MeV)	
Ref.	^{3}He η_c	^{4}He η_c	NM η_c	^{4}He J/ψ	NM J/ψ
[1]	19	140	$*$		
[2]	0.8	5	27		
[3]			10		10
[5]	$*$	$*$	9		
[6]					5
[7]				5	18
[8]				15.7	

NPLQCD

TABLE V. The binding energies (in MeV) of charmonium-nucleus systems calculated on the $L = 24$ and 32 ensembles. The rightmost column shows the infinite-volume estimate, which, without results on the $L = 48$ ensemble, is taken to be the binding calculated on the $L = 32$ ensemble. The first and second sets of parentheses shows the statistical and quadrature-combined statistical plus systematic uncertainties, respectively.

System	$24^3 \times 64$	$32^3 \times 64$	$L = \infty$
$N\eta_c$	17.9(0.4)(1.5)	19.8(0.7)(2.6)	19.8(2.6)
$d\eta_c$	39.3(1.3)(4.8)	42.4(1.1)(7.9)	42.4(7.9)
$pp\eta_c$	37.8(1.1)(4.5)	41.5(1.0)(7.5)	41.5(7.6)
^{3}Heη_c	57.2(1.3)(8.3)	56.7(2.0)(9.4)	56.7(9.6)
^{4}Heη_c	70(02)(13)	56(06)(17)	56(18)
^{4}HeJ/ψ	75.7(1.9)(9.4)	53(07)(18)	53(19)

Need crucial input

— DN interaction

Need crucial input

— DN interaction

P̄ANDA @ FAIR

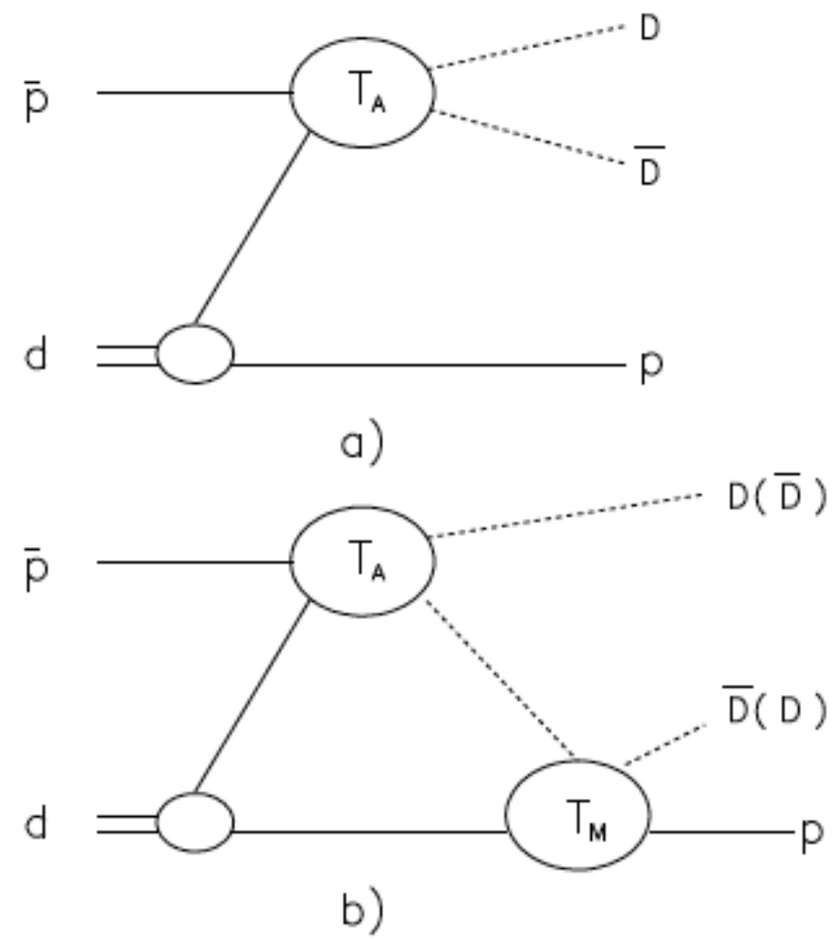
Need crucial input
— DN interaction

P̄ANDA @ FAIR

DN Experiment

- antiproton annihilation on the deuteron*

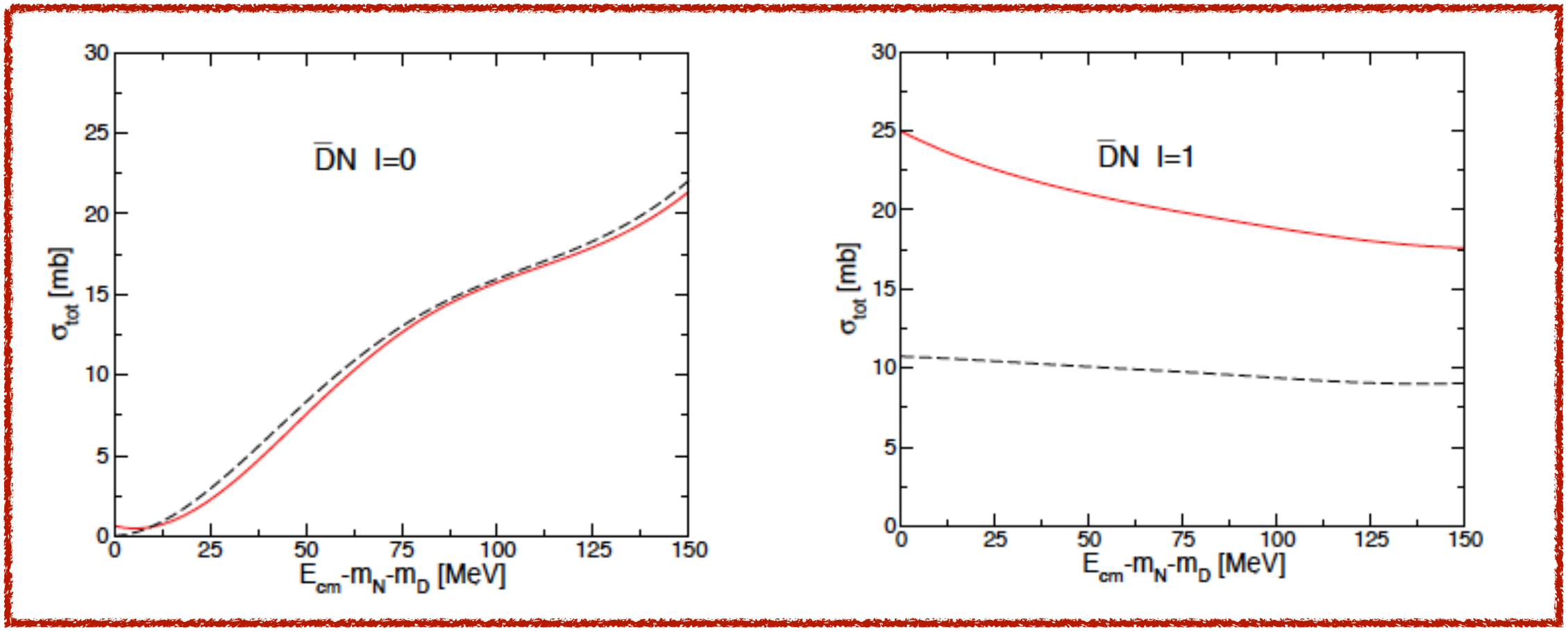
P̄anda @ FAIR



* J. Haidenbauer, GK, U.-G. Meissner, A. Sibirtsev
1) Eur. Phys. J. A 33, 107 (2007)
2) Eur. Phys. J. A 37, 55 (2008)

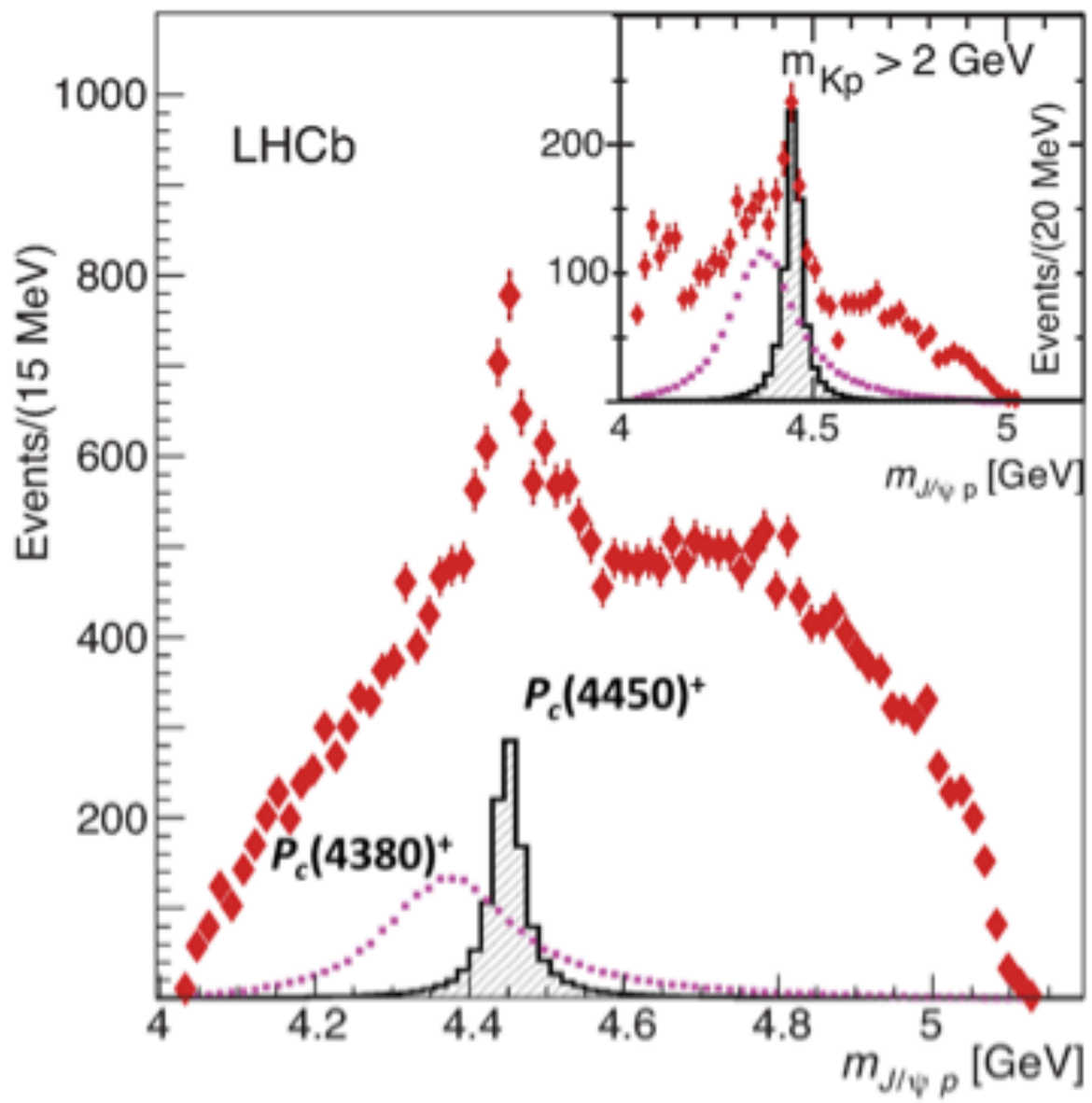
Predictions for the PANDA measurement

Use SU(4) symmetry for couplings:



Similar magnitude to KN

J/Ψ binding to proton?



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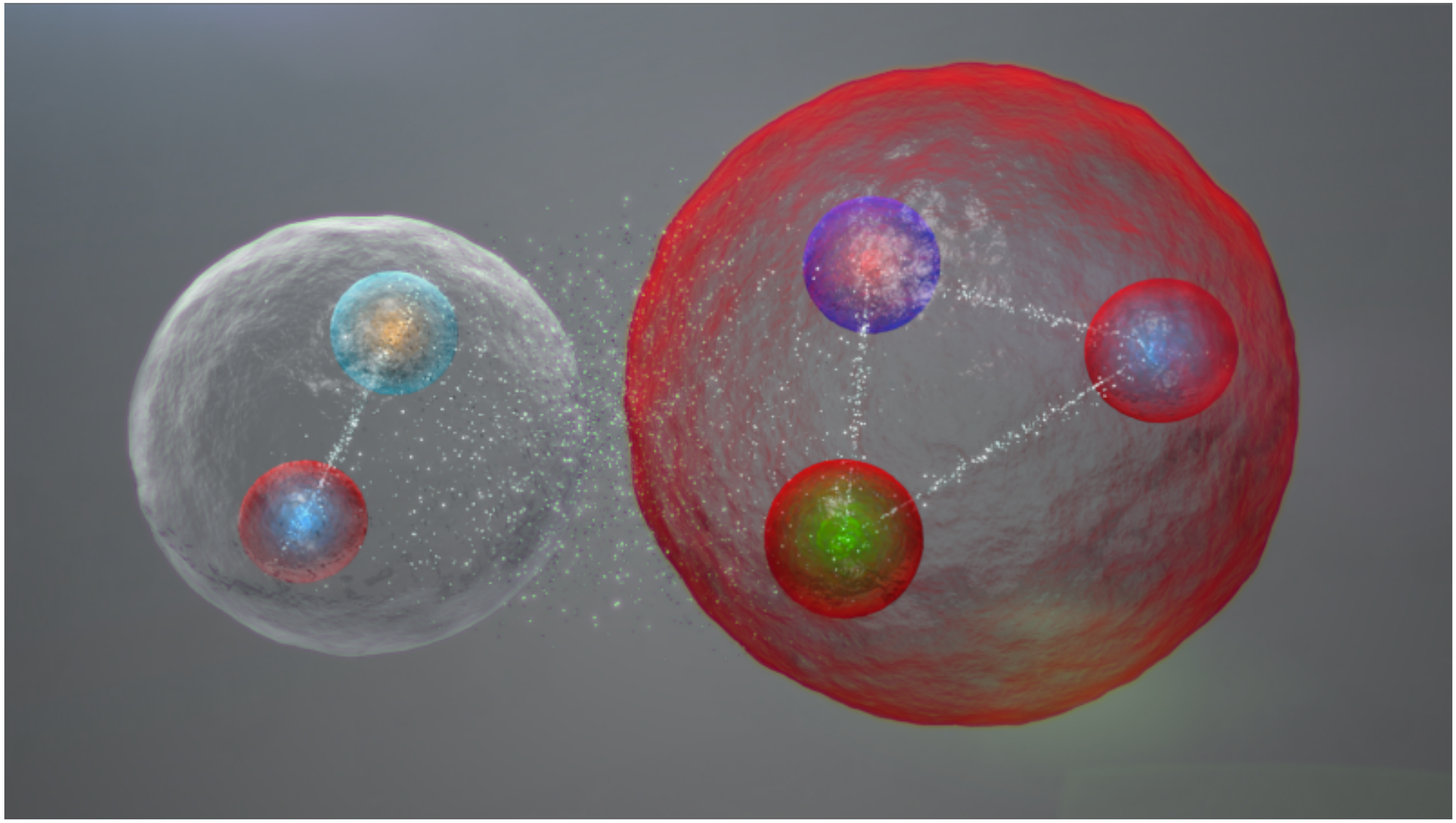

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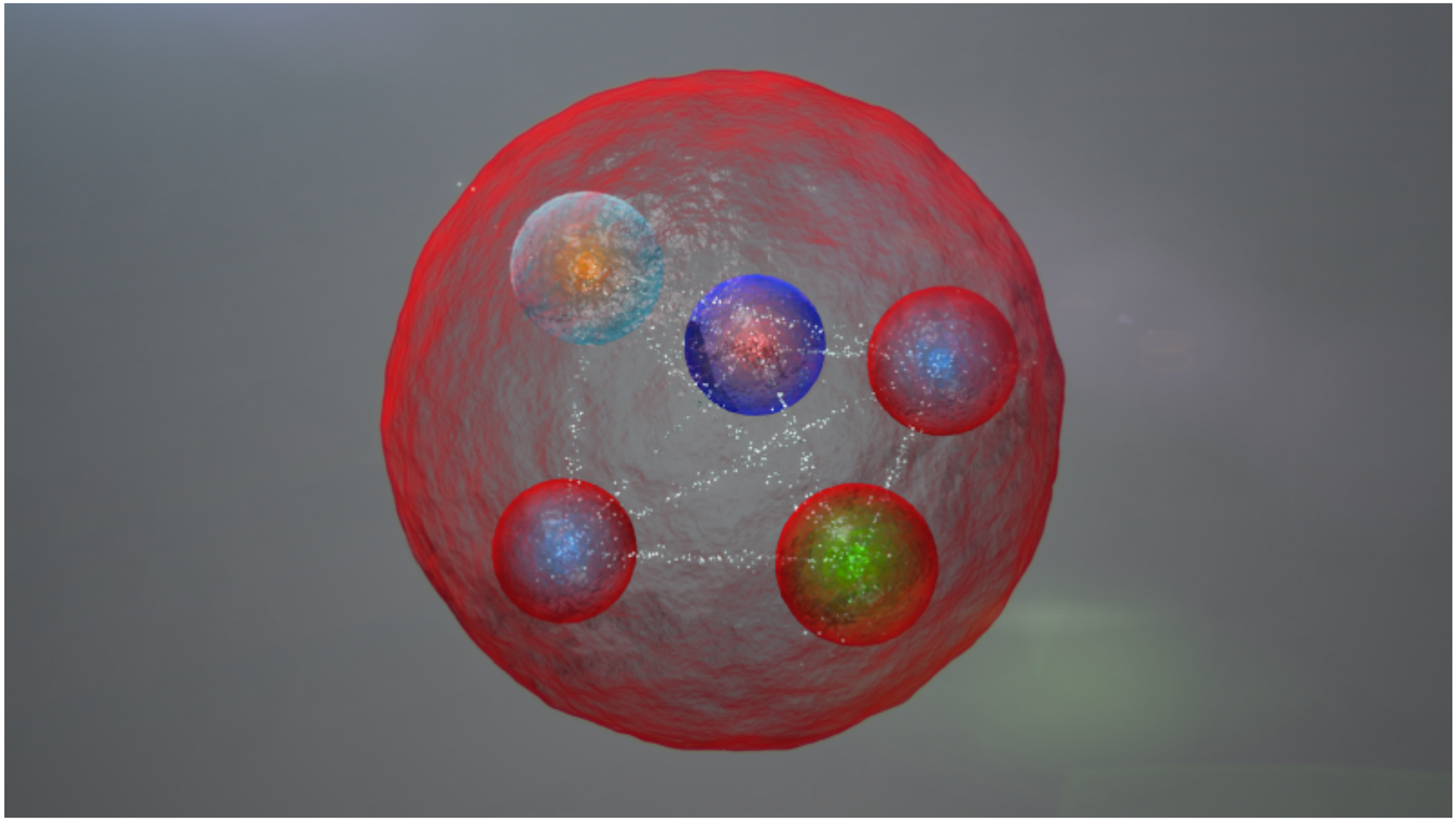

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Chromopolarizability & color van der Waals forces

— an EFT perspective

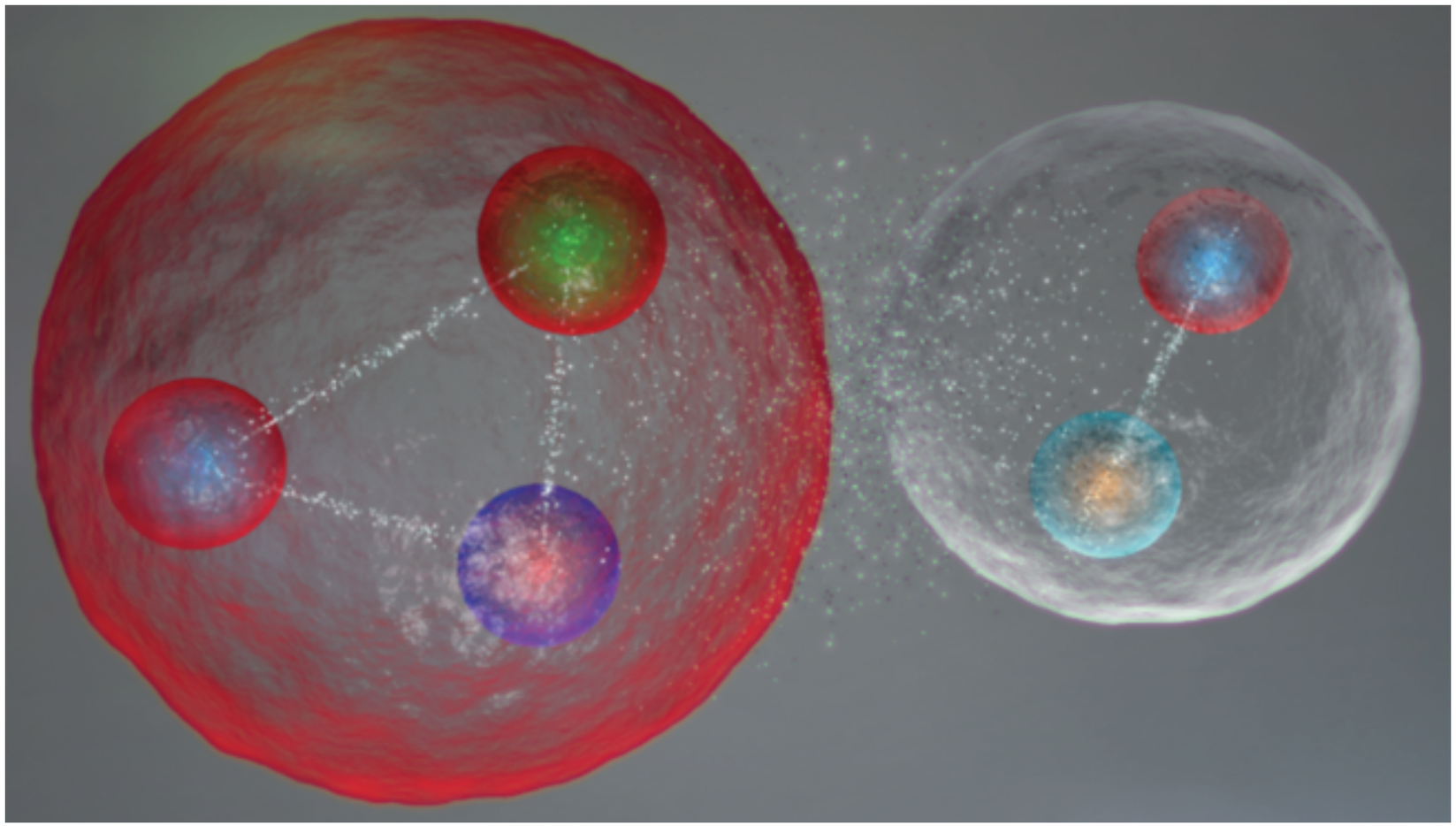
Interactions between color neutral objects:

Via creation of instantaneous color dipole moments & gluon transitions in virtual color-octet intermediate state

— Polarizability—

N. Brambilla, GK, J. Tarrús-Castellà, A. Vairo, PRD 93, 054002 (2016)

Would like to treat this



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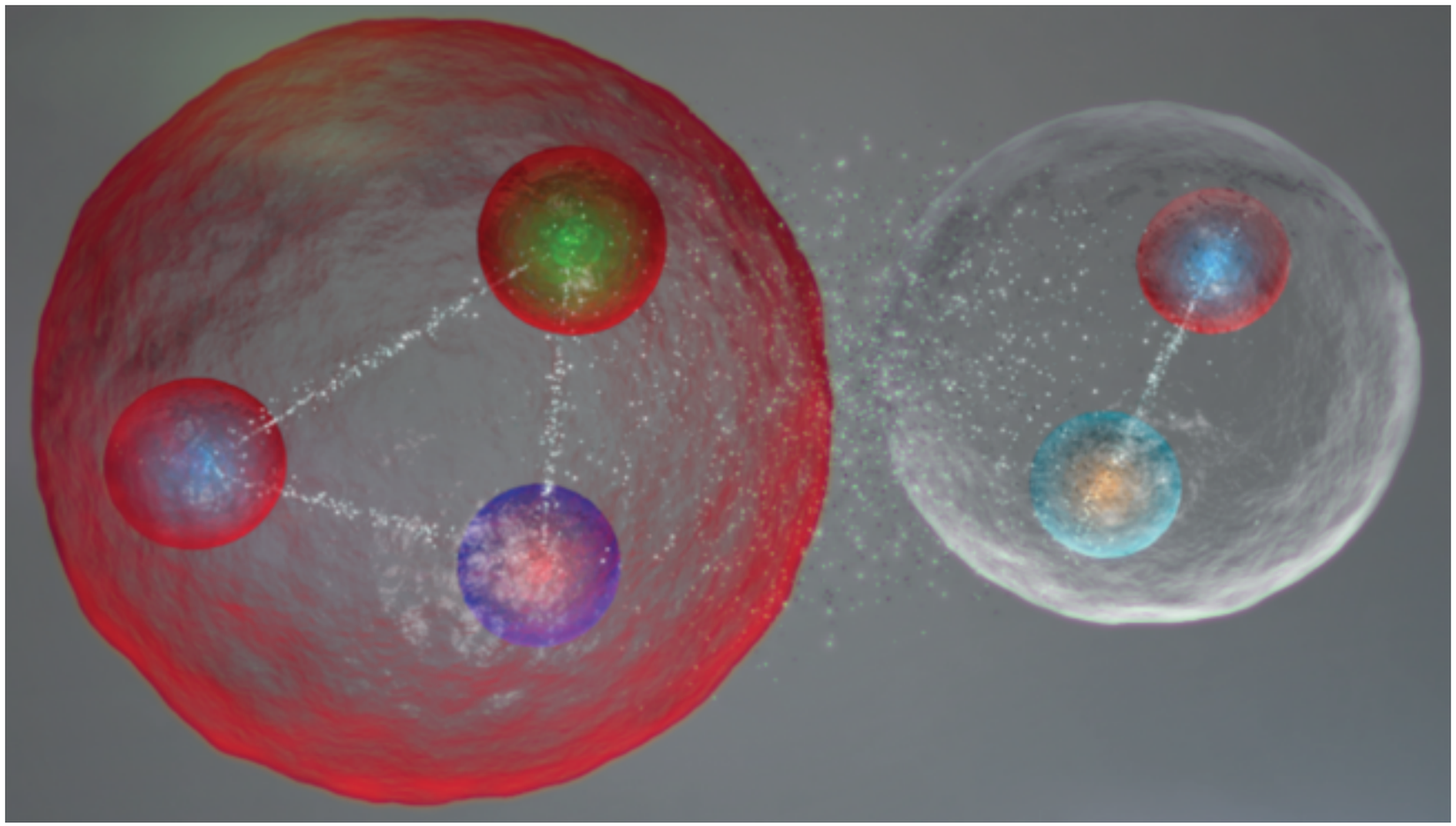
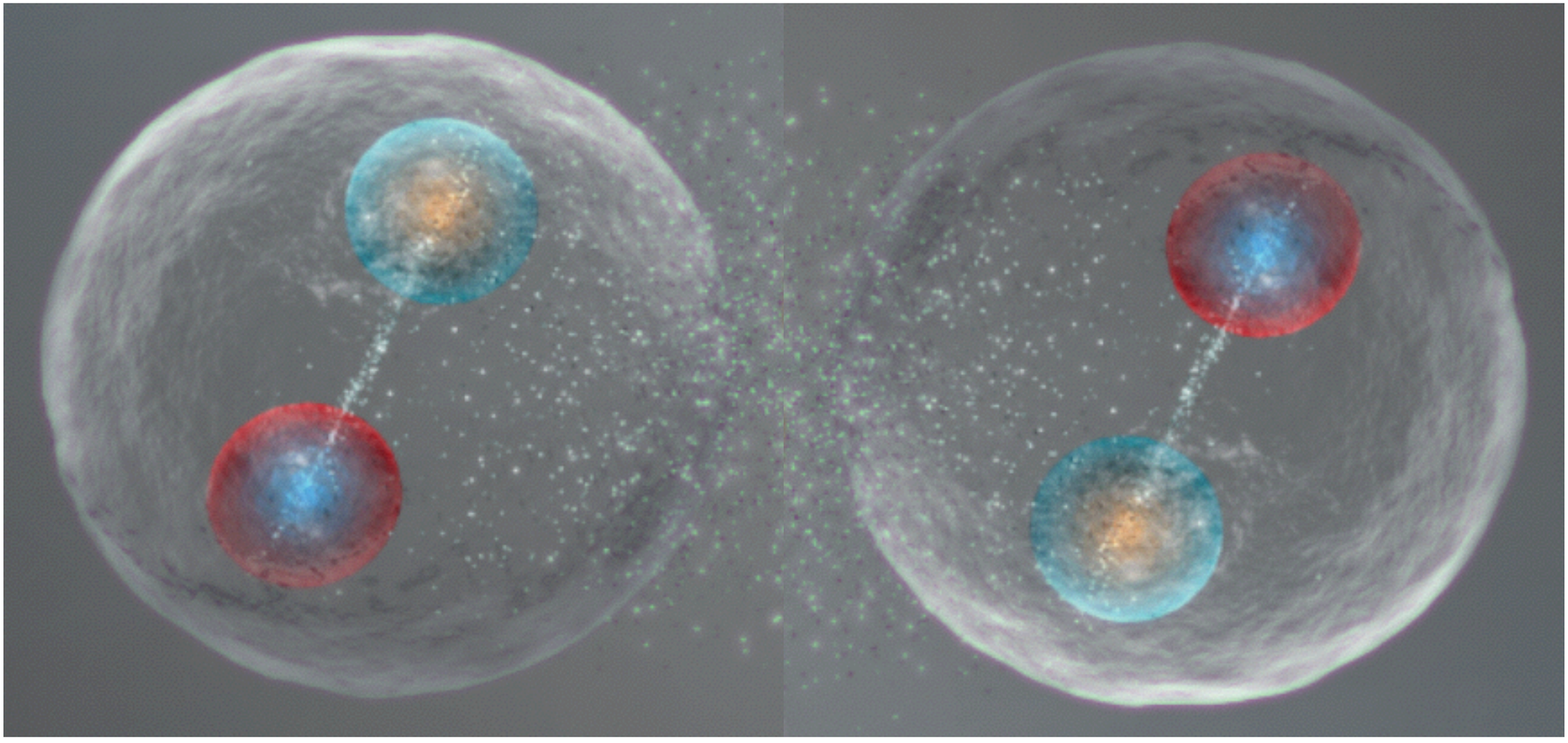


But will start with a simpler system …

$$\eta_b - \eta_b$$



EFT approch

— Chromopolarizability of 1S bottomonium;
use pNRQC (potential Nonrelativistic QCD)

— van der Waals force between two bottomonia;
use QCD trace anomaly to match pNRQC to a chiral EFT

$$\begin{tikzcd}
\text{QCD} \arrow[r] & \text{NRQCD} \arrow[r] & \text{pNRQCD} \arrow[d] & & \\
& & \text{gWEFT} \arrow[r] & \chi\text{EFT} \arrow[r] & \text{WEFT}
\end{tikzcd}$$

Polarizability van der Waals

Scales

m : bottom mass, v : relative velocity

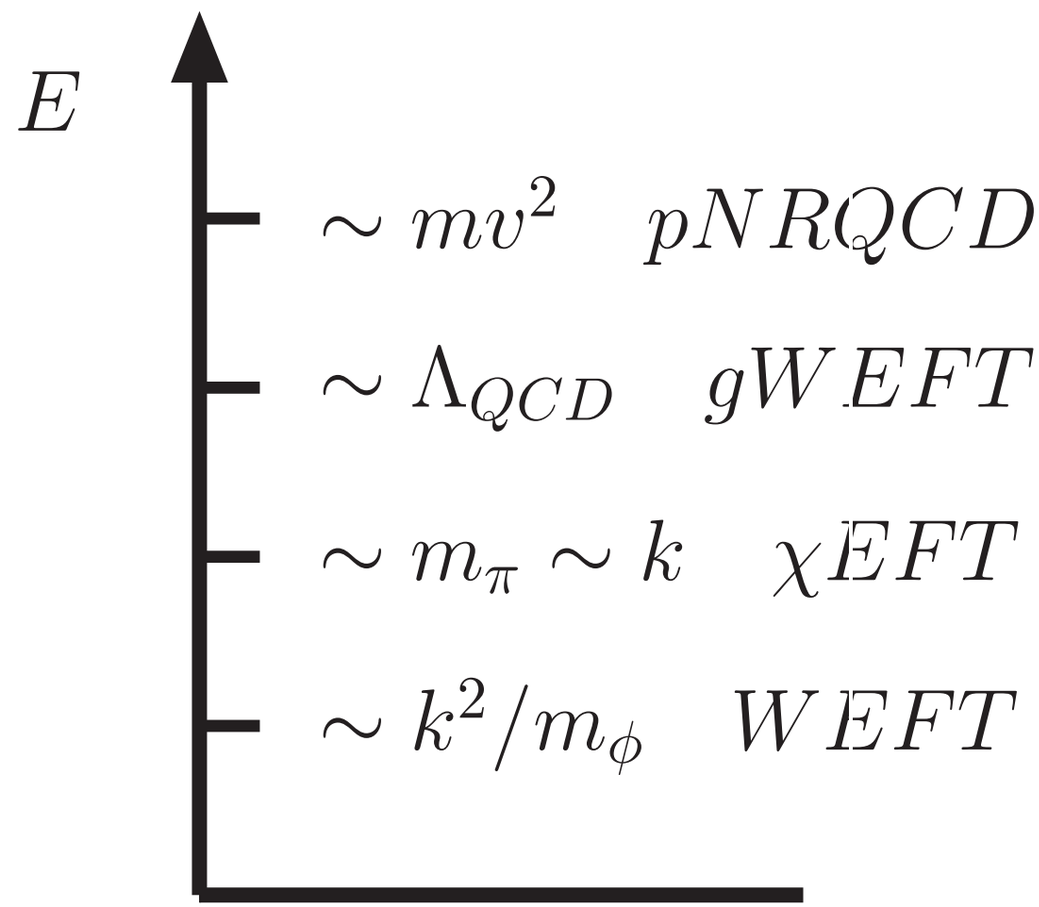
$$m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$$

QCD $\longrightarrow$ NRQCD $\longrightarrow$ pNRQCD $\longrightarrow$ gWEFT

m_ϕ : mass bottomonium, $r_{\phi\phi} \sim 1/m_\pi$: relative distance

$$\mathbf{k}^2_{\phi\phi}/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

gWEFT $\longrightarrow$ χEFT $\longrightarrow$ WEFT



Hierarchies of scales and the corresponding EFTs

A potential worry

— are the different hierarchies satisfied?

A potential worry

— are the different hierarchies satisfied?

Assume they are, see what are the consequences

— a self-consistent, systematic, modern approach

— some old (forgotten?) results reproduced

— corrected and completed early calculations

pNRQCD*

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\left[S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right]$$
$$+ gV_A(r){\rm Tr}\left[O^\dagger \boldsymbol{r}\cdot\boldsymbol{E}S + S^\dagger\boldsymbol{r}\cdot\boldsymbol{E}O\right] + \frac{g}{2}V_B(r){\rm Tr}\left[O^\dagger\boldsymbol{r}\cdot\boldsymbol{E}O + O^\dagger O\boldsymbol{r}\cdot\boldsymbol{E}\right]$$
$$+ \mathcal{L}_{\rm light}$$

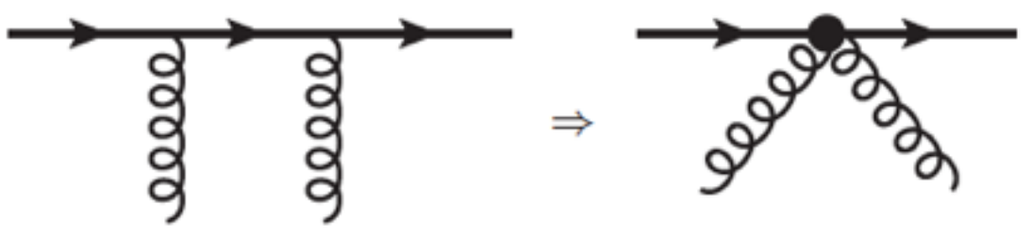
$$h_s = -\frac{\boldsymbol{\nabla}_r^2}{m} - \frac{\boldsymbol{\nabla}_R^2}{4m} + V_s(r)\,,$$
$$h_o = -\frac{\boldsymbol{\nabla}_r^2}{m} - \frac{\boldsymbol{D}_R^2}{4m} + V_o(r)\,,$$

$$V_s(r) = -C_F\frac{\alpha_s(r)}{r}\,,$$
$$V_o(r) = \left(\frac{C_A}{2} - C_F\right)\frac{\alpha_s(r)}{r}\,,$$
$$V_A(r) = 1\,,$$
$$V_B(r) = 1\,,$$

$C_A = N_c = 3$, $C_F = (N_c^2 - 1)/(2N_c)$ and $T_F = 1/2$

* A. Pineda and J. Soto, Nucl. Phys. B, Proc. Suppl. 64, 428 (1998)
N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nucl. Phys. B 566, 275 (2000)

Chromopolarizability



pNRQCD $\longrightarrow$ gWEFT

$$L_{\mathrm{gWEFT}} = \int d^3\boldsymbol{R} \left\{ \phi^\dagger(t,\boldsymbol{R}) \left[i\partial_0 + E_\phi - \frac{\boldsymbol{\nabla}_{\boldsymbol{R}}^2}{4m} + \frac{1}{2}\beta g^2 \boldsymbol{E}_a^2 + \cdots \right] \phi(t,\boldsymbol{R}) \right\} + \mathcal{L}_{\mathrm{light}}$$

Chromopolarizability

$$\beta = -\frac{2V_A^2 T_F}{3N_c} \langle \phi | \boldsymbol{r} \frac{1}{E_\phi - h_o} \boldsymbol{r} | \phi \rangle$$

$$= -\frac{2V_A^2 T_F}{3N_c} \sum_l \int \frac{d^3p}{(2\pi)^3} |\langle \phi | \boldsymbol{r} | \boldsymbol{p} l \rangle|^2 \frac{1}{E_\phi - \frac{p^2}{m}}$$

Wave functions

Bound-state

$$\langle \boldsymbol{r}|\phi\rangle = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \qquad a_0 = 1/(m\alpha_s)$$

Continuum octet*

$$|\langle \phi|\boldsymbol{r}|\boldsymbol{p}1\rangle|^2 = \frac{512\pi^2\rho(\rho+2)^2 a_0^6|\boldsymbol{p}|(1+\frac{\rho^2}{a_0^2|\boldsymbol{p}|^2})e^{\frac{4\rho}{a_0|\boldsymbol{p}|}\arctan(a_0|\boldsymbol{p}|)}}{(e^{\frac{2\pi\rho}{a_0|\boldsymbol{p}|}}-1)(1+a_0^2|\boldsymbol{p}|^2)^6}$$

$$\rho = (N_c^2-1)^{-1}$$

Results: polarizability

$$\beta = 256 \frac{\rho(\rho+2)^2}{3N_c} \frac{1}{mE_\phi^2} I$$

$$\rho = (N_c^2 - 1)^{-1}$$

$$I = \int_0^\infty dp\, p^3 \frac{(1+\frac{\rho^2}{p^2}) e^{\frac{4\rho}{p}\arctan p}}{(e^{\frac{2\pi\rho}{p}} - 1)(1+p^2)^7} \underset{N_c=3}{=} 0.01143$$

Agrees with:

H. Leutwyler, Phys. Lett. **98B**, 447 (1981)

M. B. Voloshin, Yad. Fiz. **36**, 247 (1982)

Results: polarizability

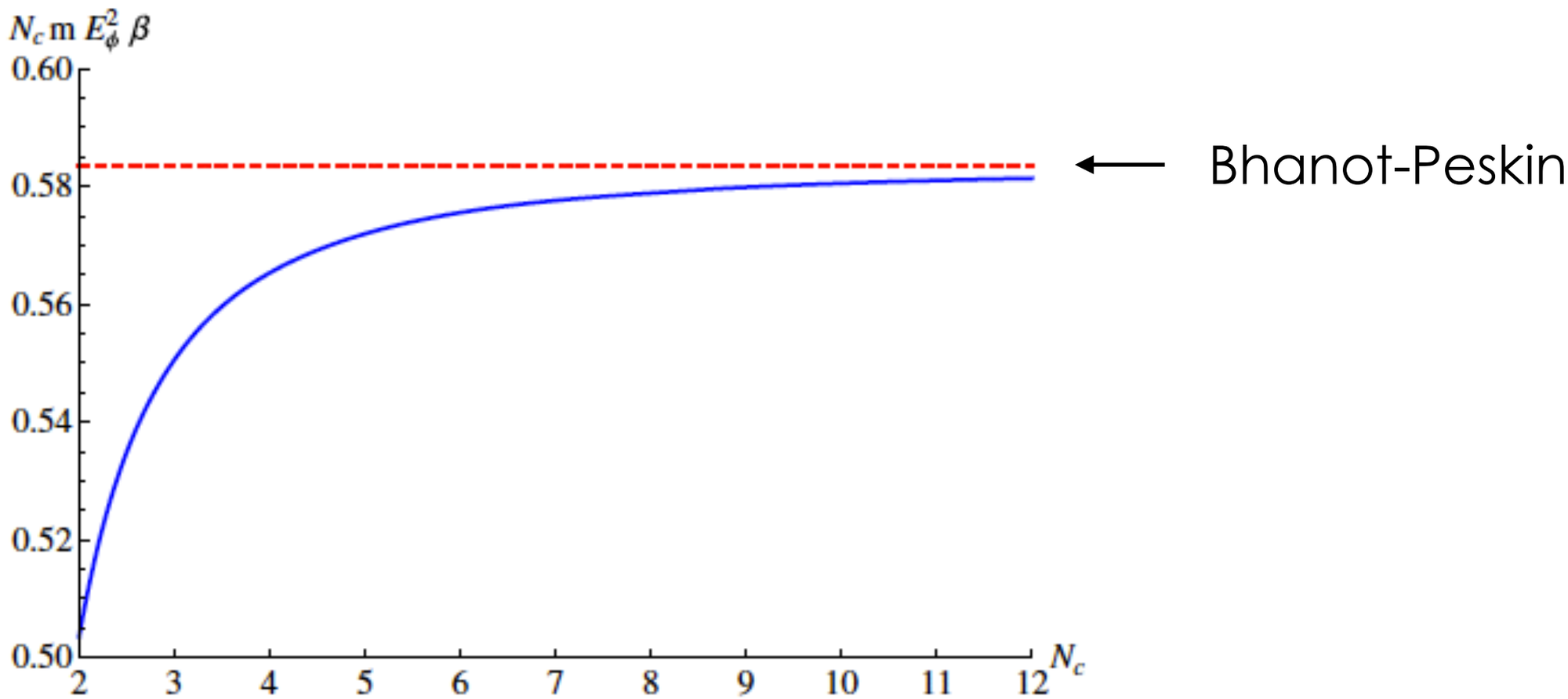


FIG. 3. The dependence of the polarizability on the number of colors. The dashed line at the constant value 7/12 corresponds to the large-N_c limit computed in Ref. [13].

Results: polarizability

$$E_\phi = -m\,\frac{(C_F\alpha_s)^2}{4} = -\frac{1}{ma_0^2}$$

$$m_\phi = 9.4454 \text{ GeV} \left\{ \begin{array}{l} \text{average of} \\ \eta_b \ \ \& \ \ \Upsilon_b(1S) \end{array} \right.$$

$$m = 5 \text{ GeV}$$

$$\alpha_s(1 \text{ GeV}) \approx 0.5$$

$$\alpha_s(1.5 \text{ GeV}) \approx 0.35$$

$$\alpha_s(2 \text{ GeV}) \approx 0.3$$

$$\beta = 0.50^{+0.42}_{-0.38} \text{ GeV}^{-3}$$

$$\Upsilon(2S) \to \Upsilon(1S)\pi\pi$$

$$\beta_{\Upsilon-\Upsilon'} = 0.66 \text{ GeV}^{-3}$$

M.B. Voloshin, Mod. Phys. Lett A 19, 665 (2004)

Results: polarizability

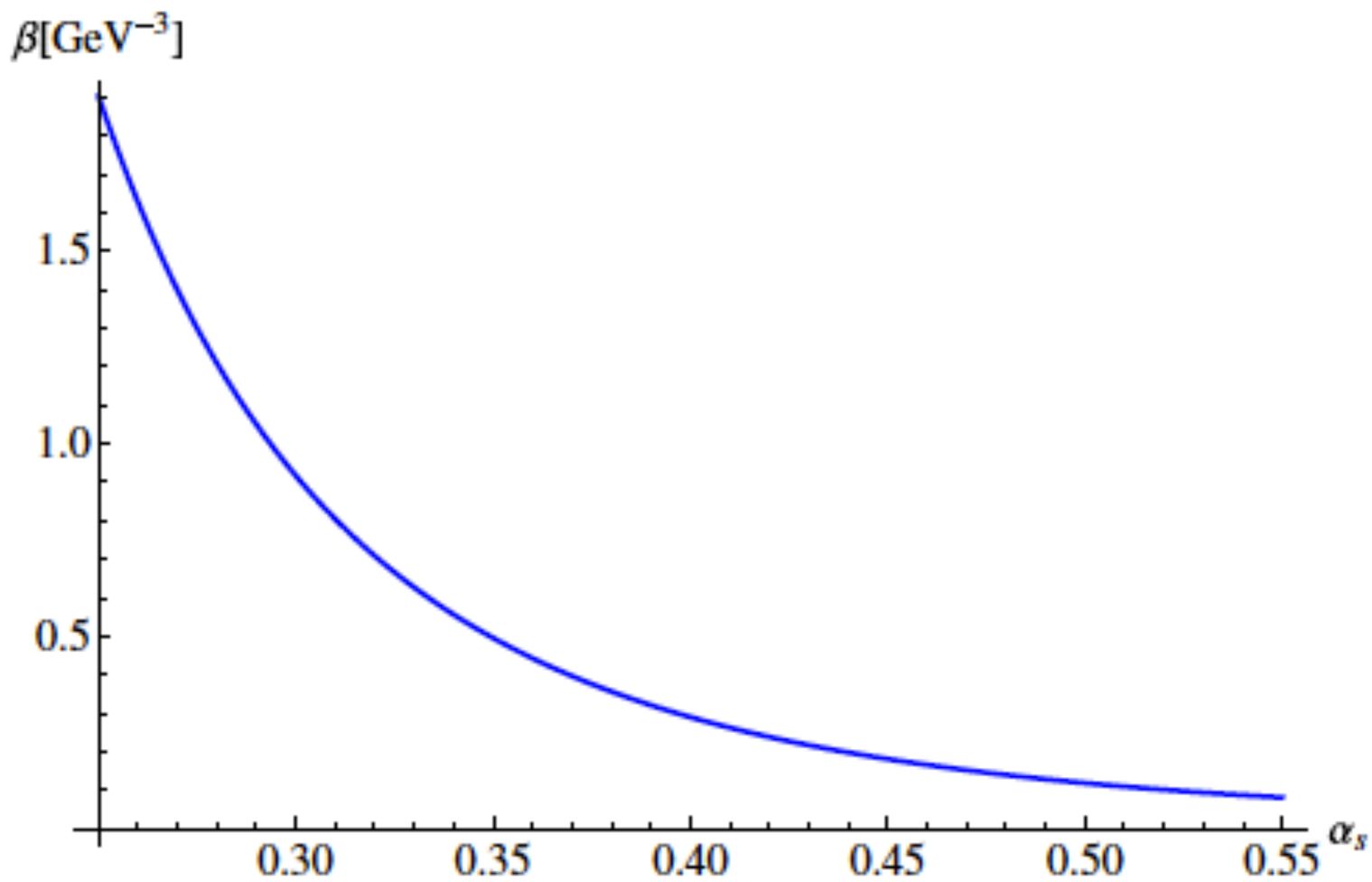
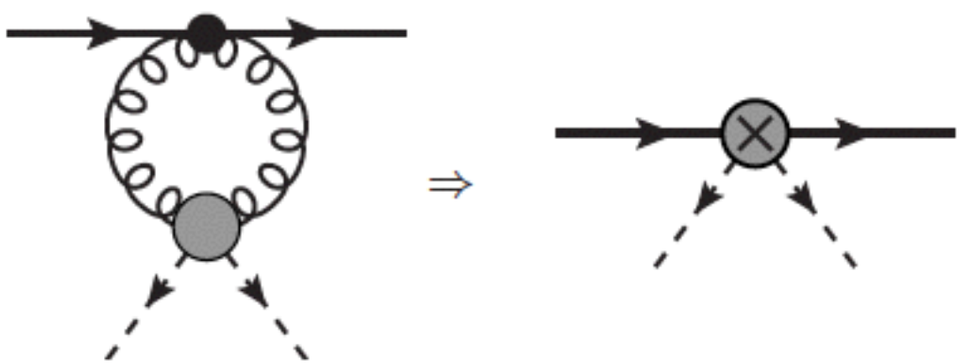


FIG. 4. Plot of β from Eq. (12) as a function of α_s for $m = 5$ GeV.

van der Waals force

gWEFT $\longrightarrow$ χEFT



QCD trace anomaly

$$g^2\langle\pi^+(p_1)\pi^-(p_2)|\boldsymbol{E}_a^2|0\rangle = \frac{8\pi^2}{3b}\left((p_1+p_2)^2\kappa_1 + m_\pi^2\kappa_2\right)$$

$$\kappa_1 = 1 - 9\kappa/4,\ \kappa_2 = 1 - 9\kappa/2 \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$\kappa = 0.186 \pm 0.003 \pm 0.006 \quad \longleftarrow \quad \psi' \to J/\psi\pi^+\pi^-$$

Matching gWEFT $\longrightarrow$ χEFT

$$\mathcal{L}^{\phi}_{\chi\mathrm{EFT}} = \phi^\dagger \left(i\partial_0 - \frac{\boldsymbol{\nabla}^2}{2m_\phi} \right) \phi$$

$$U = e^{i\phi/F} = u^2\,, \quad \phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\mathcal{L}^{\pi}_{\chi\mathrm{EFT}} = \frac{F^2}{4} \left(\langle \partial_\mu U \partial^\mu U^\dagger \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \right)$$

$$\chi = 2B\hat{m}\mathbf{1} \qquad F = F_\pi = 92.419\,\mathrm{MeV}$$

$$\mathcal{L}^{\phi-\pi}_{\chi\mathrm{EFT}} = \phi^\dagger \phi \frac{F^2}{4} \left(c_{d0} \langle \partial_0 U \partial_0 U^\dagger \rangle + c_{di} \langle \partial_i U \partial^i U^\dagger \rangle + c_m \langle \chi^\dagger U + \chi U^\dagger \rangle \right)$$

$$\boxed{\mathcal{A} = \frac{4\pi^2\beta}{b} \left(\kappa_1 p_1^0 p_2^0 - \kappa_2 p_1^i p_2^i + 3m_\pi^2 \right)}$$

$$\boxed{\mathcal{A} = -c_{d0} p_1^0 p_2^0 + c_{di} p_1^i p_2^i - c_m m_\pi^2}$$

$$c_{d0} = -\frac{4\pi^2\beta}{b}\kappa_1 \qquad c_{di} = -\frac{4\pi^2\beta}{b}\kappa_2 \qquad c_m = -\frac{12\pi^2\beta}{b}$$

Chiral correction

— bottomonium mass

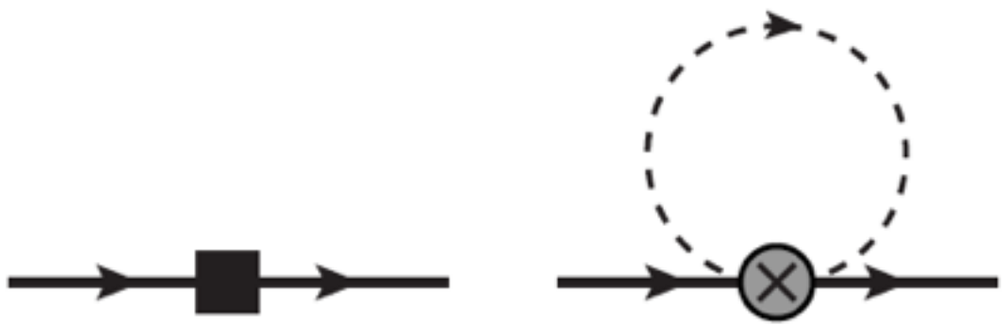
$$\delta m_\phi = -F^2 c_m m_\pi^2 + \text{counterterms of } \mathcal{O}(m_\pi^4)$$
$$+ \frac{3m_\pi^2}{8}(c_{d0} + 3c_{di} - 4c_m)A[m_\pi^2] + \frac{3m_\pi^4(c_{d0} - c_{di})}{256\pi^2}$$

$$A[m_\pi^2] = \frac{m_\pi^2}{16\pi^2}\left(\lambda - \log\frac{m_\pi^2}{\nu^2}\right)$$

$$\lambda = \frac{2}{4-d} + 1 - \gamma_E + \log 4\pi$$

Bottomonium mass

— leading chiral log contribution



$$\delta m_\phi\big|_{\text{chiral log}} = -\frac{15}{8}\frac{\beta}{b} m_\pi^4 \log\frac{m_\pi^2}{\nu^2}$$

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

Corrected
Grinstein & Rothstein PLB 385, 265 (1996)

van der Waals force

$$r_{\phi\phi} \sim 1/m_\pi \qquad\qquad \mathbf{k}^2_{\phi\phi}/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

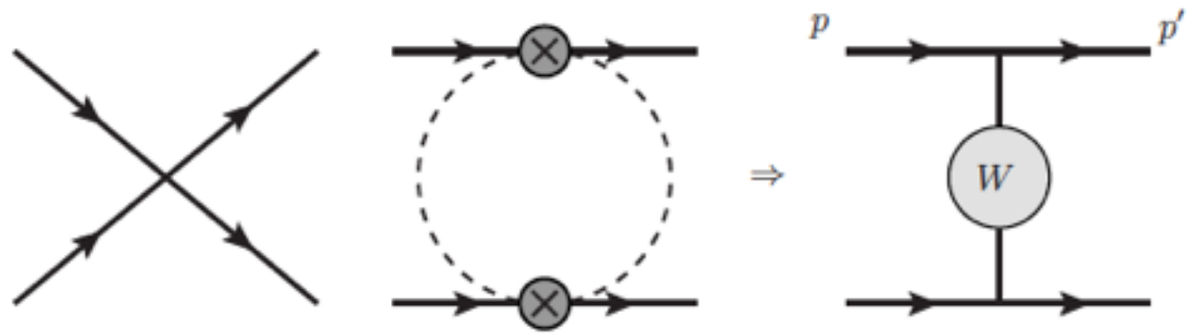
Relative motion at energies lower than pion mass

— integrate out the pion

χEFT $\longrightarrow$ WEFT

Matching

χEFT $\longrightarrow$ WEFT



$$L^{\phi}_{\mathrm{WEFT}} = \int d^3\boldsymbol{R}\phi^\dagger(t,\boldsymbol{R})\left(i\partial_0 + \frac{\nabla^2}{2m_\phi}\right)\phi(t,\boldsymbol{R})$$

$$L^{\phi\phi}_{\mathrm{WEFT}} = -\frac{1}{2}\int d^3\boldsymbol{R}_1 d^3\boldsymbol{R}_2 \phi^\dagger\phi(t,\boldsymbol{R_1})W(\boldsymbol{R_1},\boldsymbol{R_2})\phi^\dagger\phi(t,\boldsymbol{R_2})$$

vdW potential

$$
\begin{aligned}
\widetilde{W}(\mathbf{k}^2) \quad = \quad & \text{contact terms} \\
& -\frac{3}{8}\left(\mathbf{k}^2 c_{di} + 2m_\pi^2\left(c_{di} - c_m\right)\right)^2 \; B\left[m_\pi^2, -\mathbf{k}^2\right] \\
& -\frac{3}{2}\left(c_{d0} - c_{di}\right)\left(\mathbf{k}^2 c_{di} + 2m_\pi^2\left(c_{di} - c_m\right)\right) \; C_1\left[m_\pi^2, -\mathbf{k}^2\right] \\
& -\frac{3}{2}\left(c_{d0} - c_{di}\right)^2 \; C_2\left[m_\pi^2, -\mathbf{k}^2\right]
\end{aligned}
$$

$$B\left[m_\pi^2\,,-\mathbf{k}^2\right]=\frac{1}{16\pi^2}\left(\lambda+1-\log\frac{m_\pi^2}{\nu^2}+\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$B\left[m_\pi^2\,,-\mathbf{k}^2\right]=\frac{1}{16\pi^2}\left(\lambda+1-\log\frac{m_\pi^2}{\nu^2}+\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$\begin{aligned}C_2\left[m_\pi^2\ -\mathbf{k}^2\right] \quad &= \quad \frac{31\mathbf{k}^4+280\mathbf{k}^2m_\pi^2+705m_\pi^4}{19200\pi^2}+\frac{1}{1280\pi^2}\Bigg(\left(\mathbf{k}^4+10\mathbf{k}^2m_\pi^2+30m_\pi^4\right)\\ &\times\left(\lambda-\log\frac{m_\pi^2}{\nu^2}\right)+\mathbf{k}^4\left(1+\frac{4m_\pi^2}{\mathbf{k}^2}\right)^{5/2}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\Bigg)\end{aligned}$$

$$B\left[m_\pi^2\,,-\mathbf{k}^2\right] = \frac{1}{16\pi^2}\left(\lambda + 1 - \log\frac{m_\pi^2}{\nu^2} + \sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$\begin{aligned} C_2\left[m_\pi^2\ -\mathbf{k}^2\right] \quad = \quad & \frac{31\mathbf{k}^4+280\mathbf{k}^2m_\pi^2+705m_\pi^4}{19200\pi^2} + \frac{1}{1280\pi^2}\Bigg(\left(\mathbf{k}^4+10\mathbf{k}^2m_\pi^2+30m_\pi^4\right) \\ & \times\left(\lambda-\log\frac{m_\pi^2}{\nu^2}\right) + \mathbf{k}^4\left(1+\frac{4m_\pi^2}{\mathbf{k}^2}\right)^{5/2}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\Bigg) \end{aligned}$$

$$\begin{aligned} C_1\left[m_\pi^2\ -\mathbf{k}^2\right] \quad = \quad & \frac{5\mathbf{k}^2+24m_\pi^2}{576\pi^2} + \frac{1}{192\pi^2}\Bigg(\left(\mathbf{k}^2+6m_\pi^2\right)\left(\lambda-\log\frac{m_\pi^2}{\nu^2}\right) \\ & + \mathbf{k}^2\left(1+\frac{4m_\pi^2}{\mathbf{k}^2}\right)^{3/2}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\Bigg) \end{aligned}$$

Final result

— vdW potential

$$W(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\, \mu\, e^{-\mu r}\, \mathrm{Im}\left[\widetilde{W}(\epsilon - i\mu)\right]$$

$$= -\frac{3\pi\beta^2 m_\pi^2}{8b^2 r^5}\Big[\Big(4\left(\kappa_2+3\right)^2 (m_\pi r)^3 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right) m_\pi r\Big) K_1(2m_\pi r)$$

$$+2\left(2\left(\kappa_2+3\right)\left(\kappa_1+5\kappa_2\right)(m_\pi r)^2 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right)\right) K_2(2m_\pi r)\Big]$$

↓ asymptotic

$$W(r) = -\frac{3(3+\kappa_2)^2\pi^{3/2}\beta^2}{4b^2}\frac{m_\pi^{9/2}}{r^{5/2}}e^{-2m_\pi r}$$

Completed the calculation of
H. Fujii and D. Kharzeev, PRD 60, 114039 (1999)

Numerical result

— vdW potential

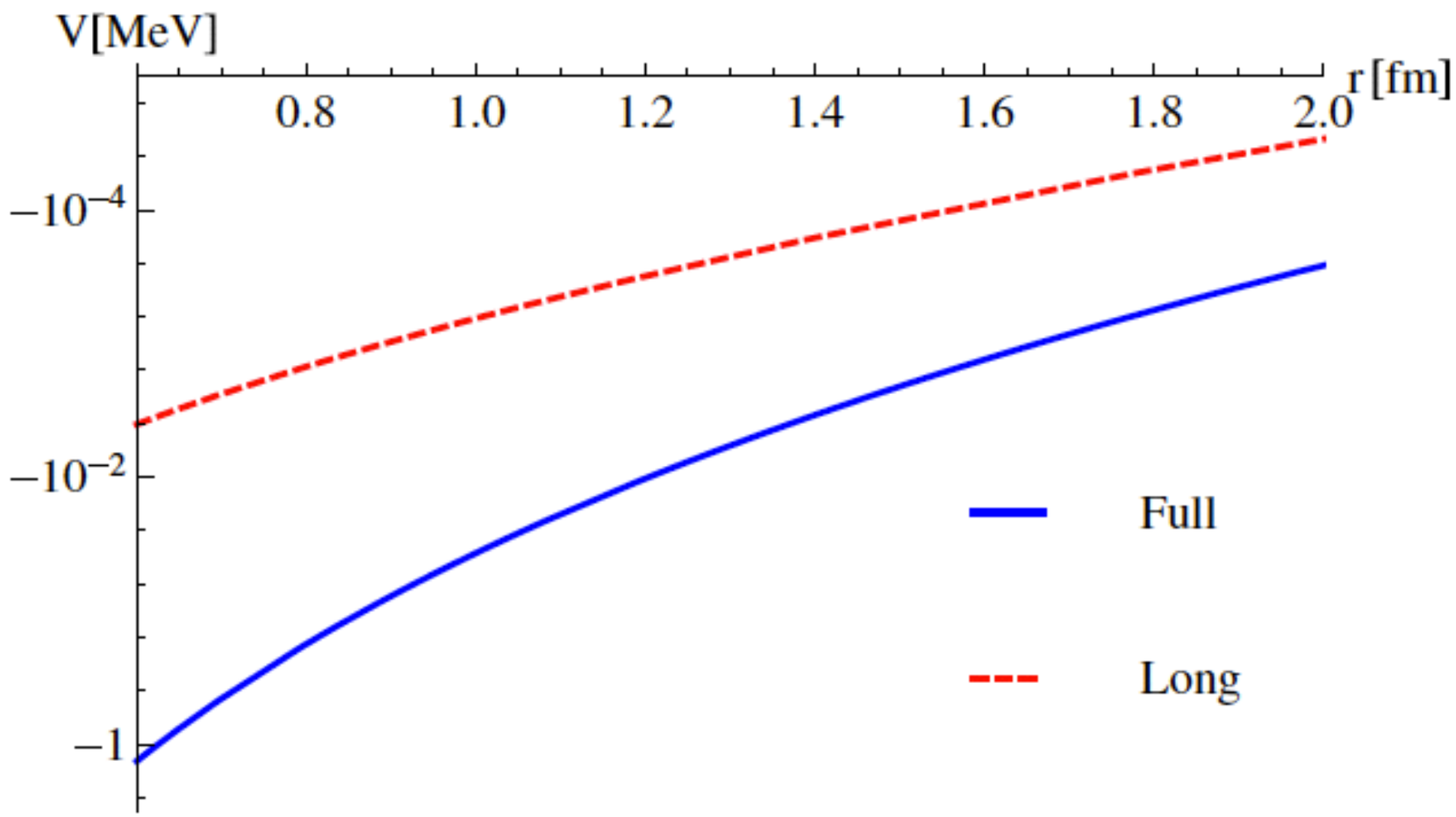


FIG. 9. Comparison of the van der Waals potential (40) (blue line) with its long-range expansion (41) (red line) for $\beta = 0.92\ \mathrm{GeV}^{-3}$ and other parameters like in Fig. 8.

Are there $\eta_b\,\eta_b$ bound-states?

— It is likely, but depends somewhat on the medium- and short-range pieces

Perspectives

— Nonperturbative input from the lattice

— Phenomenology, need experiments, e.g. DN

— EFT for molecules, Born-Oppenheimer

(see talk of J. Tarrús)

Funding

FAPESP
SÃO PAULO RESEARCH FOUNDATION

BAYLAT

CNPq
Conselho Nacional de Desenvolvimento
Científico e Tecnológico