doubly heavy tetraquarks
(and other hadrons)
from lattice QCD

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outline

- motivation
- hadron operators
- lattice method
- lattice results

motivation

Lattice QCD provides precise calculations of the hadron spectrum.

Building on successes for conventional mesons and baryons,
we are exploring a particular tetraquark possibility.

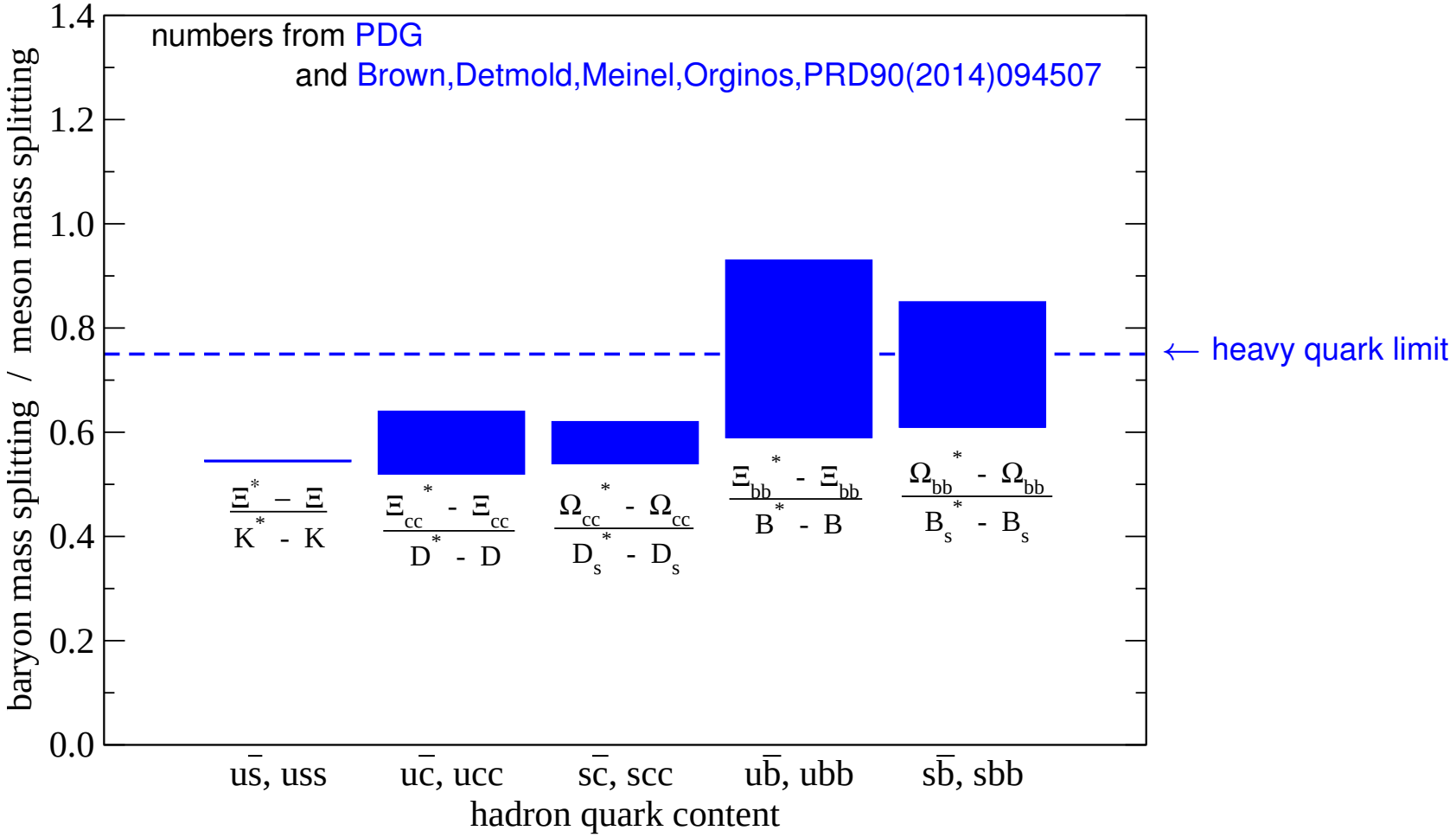
Quark content $qq\bar{Q}\bar{Q}$ avoids two major lattice difficulties:

- It needs no disconnected diagrams.
- A tetraquark would be the lightest state in this channel.

NOTE: $\bar{Q}\bar{Q}$ can be $\bar{b}\bar{b}$, $\bar{c}\bar{c}$ or $\bar{b}\bar{c}$.
qq can be uu, dd, ss, ud, us or ds.

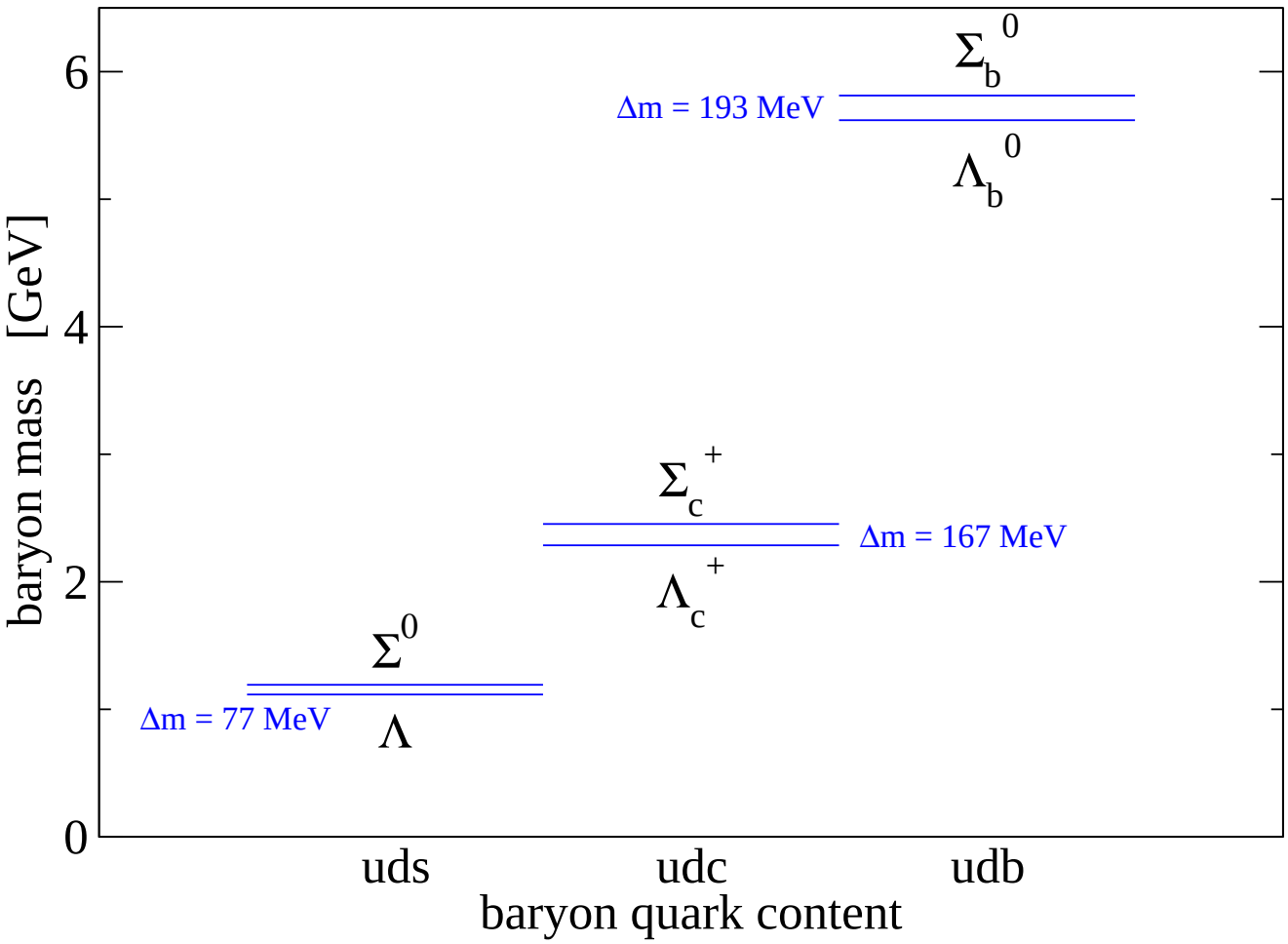
heavy diquarks are quark-like

Heavy quark symmetry relates a baryon's QQ to a meson's $\bar{Q}$.
This is observed in experiment and lattice data.



light diquarks prefer spin zero

The ud diquark binds more strongly in spin 0 (Λ) than spin 1 (Σ).



a candidate tetraquark operator

For definiteness, begin with quark content $ud\bar{b}\bar{b}$.

The ud portion should be Λ-like, not Σ-like.
Therefore ud is antisymmetric in both color and flavor:

$$L_a(x) = \epsilon_{abc} u_b^T(x) C\gamma_5 d_c(x)$$

The $\bar{b}\bar{b}$ portion will be quark-like.
To join with ud, it must be color antisymmetric but flavor symmetric:

$$H_{ai}(x) = \epsilon_{ade} \bar{b}_d(x) C\gamma_i \bar{b}_e^T(x)$$

The total operator is $O_i(x) = L_a(x) H_{ai}(x)$.

The lightest state will therefore be $\boxed{J^P = 1^+}$.

a meson-pair operator

Again, consider quark content $ud\bar{b}\bar{b}$ with $J^P = 1^+$.

The lightest conventional state would be a meson pair:
B(5279) with $J^P = 0^-$ and
B*(5325) with $J^P = 1^-$.

An operator for this pair is $P_i(x) = \bar{b}_a(x)\gamma_5 u_a(x)\bar{b}_b(x)\gamma_i d_b(x)$
It mixes with $O_i(x)$ but differs in its internal color structure.

Since $O_i(x)$ and $P_i(x)$ have the same quantum numbers, they can propagate the same physical states, though the strength of their overlaps will differ.

relativistic u,d,s,c quarks

- u, d and s quarks have familiar "clover" covariant derivative:

$$
\begin{aligned}
D(x,y) &= \delta_{x,y}\left(1-\kappa c_{SW}\sum_{\mu,\nu}F_{\mu\nu}\sigma_{\mu\nu}\right) \\
&\quad -\kappa\sum_{\mu}\left((1-\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},y}+(1+\gamma_\mu)U^{\dagger}_{x,\mu}\delta_{x,y+\hat{\mu}}\right)
\end{aligned}
$$

- relativistic c quarks (tuned by Ikeda et al, Phys Lett B729 (2014) 85):

$$
\begin{aligned}
D(x,y) &= \delta_{x,y}\left(1-\kappa c_E\sum_{i}F_{i4}\sigma_{i4}-\kappa c_B\sum_{i,j}F_{ij}\sigma_{ij}\right) \\
&\quad -\kappa\sum_{\mu}\left((r_\mu-\nu\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},y}+(r_\mu+\nu\gamma_\mu)U^{\dagger}_{x,\mu}\delta_{x,y+\hat{\mu}}\right)
\end{aligned}
$$

non-relativistic b quarks

- b quarks use a tadpole-improved NRQCD Hamiltonian:

$$
\begin{aligned}
H \quad = \quad & -\frac{\Delta^{(2)}}{2M_0} - c_1 \frac{(\Delta^{(2)})^2}{8M_0^3} + \frac{c_2}{U_0^4}\frac{ig}{8M_0^2}(\tilde{\boldsymbol{\Delta}}\cdot\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{E}}\cdot\tilde{\boldsymbol{\Delta}}) \\
& -\frac{c_3}{U_0^4}\frac{g}{8M_0^2}\boldsymbol{\sigma}\cdot(\tilde{\boldsymbol{\Delta}}\times\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{E}}\times\tilde{\boldsymbol{\Delta}}) - \frac{c_4}{U_0^4}\frac{g}{2M_0}\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{B}} \\
& +c_5\frac{a^2\Delta^{(4)}}{24M_0} - c_6\frac{a(\Delta^{(2)})^2}{16nM_0^2}
\end{aligned}
$$

lattice ensembles

We use 3 ensembles produced by the PACS-CS collaboration.

Aoki et al, Phys Rev D79, 034503 (2009)

These include dynamical u, d, s quarks.

ensemble label	E4	E5	E6
lattice size	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 64$
lattice spacing	0.09 fm	0.09 fm	0.09 fm
dynamical quarks	u, d, s	u, d, s	u, d, s
# of configs	400	800	198
pion mass	410 MeV	298 MeV	156 MeV

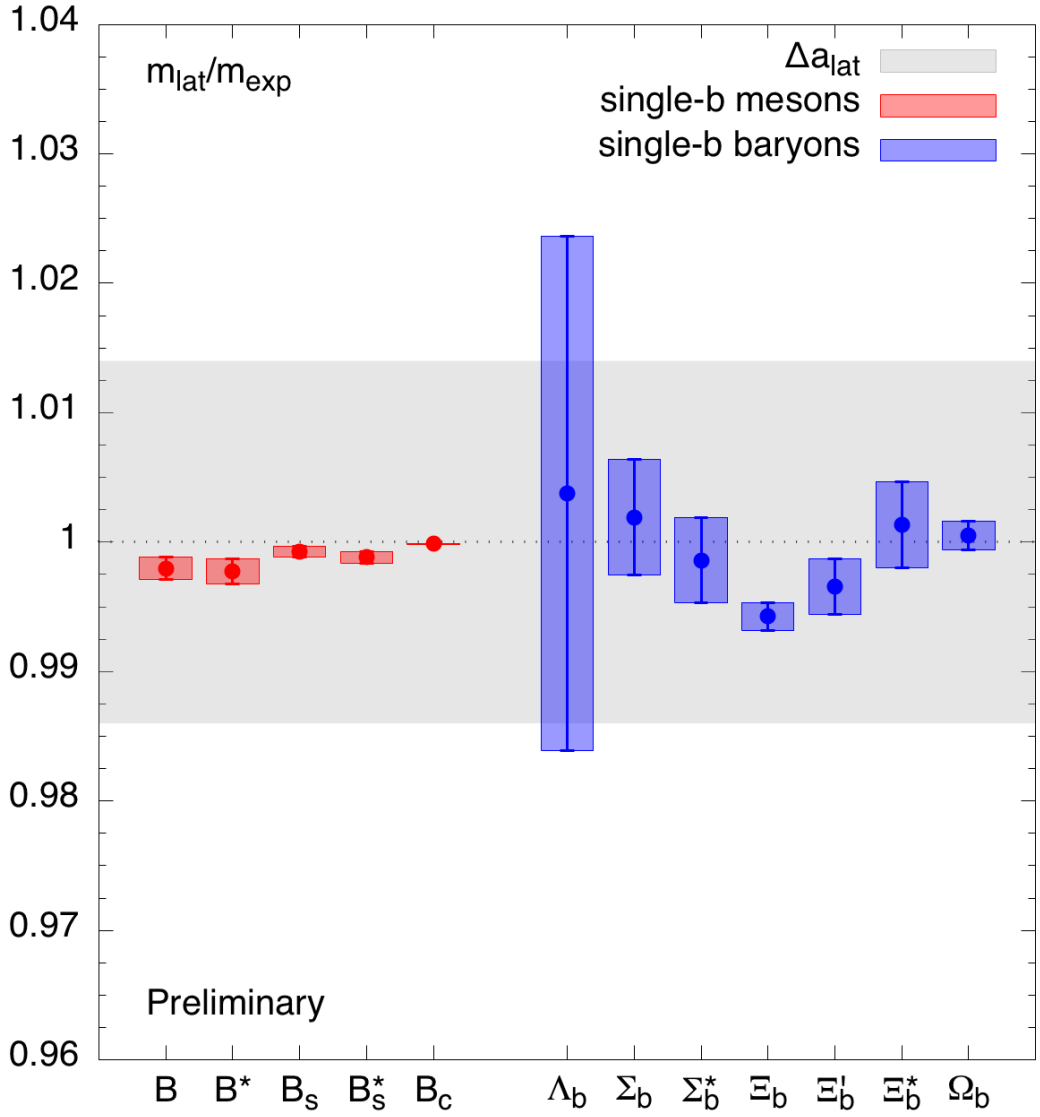
We use multiple sources per config to attain
≥ 800 measurements for every hadron mass we compute.

We use gauge-fixed wall sources to reduce statistical errors.

confirming the basic spectrum

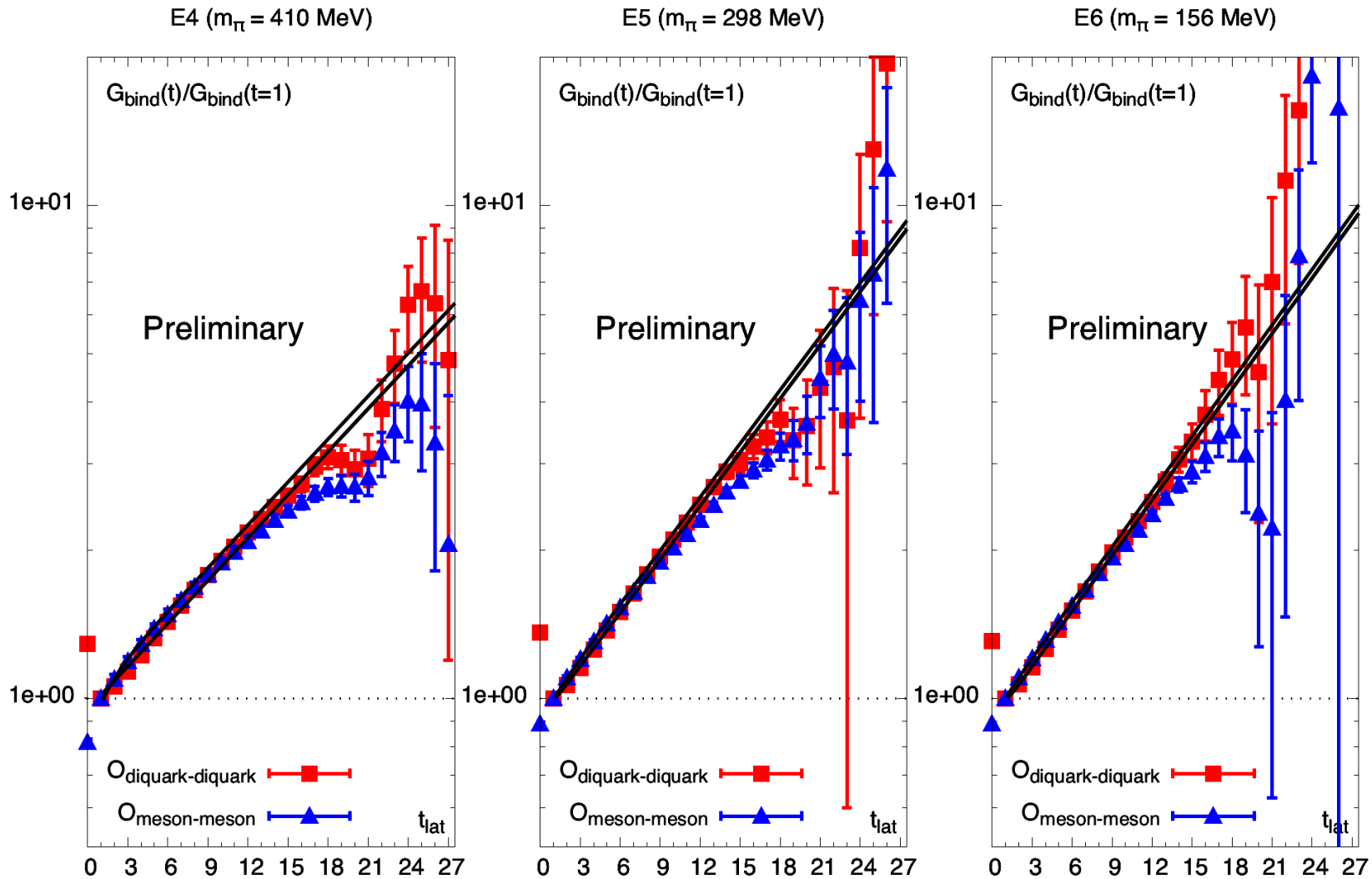
Our lattice set-up is tested by computing several heavy hadrons.

These results come from a chiral extrapolation of our 3 ensembles.

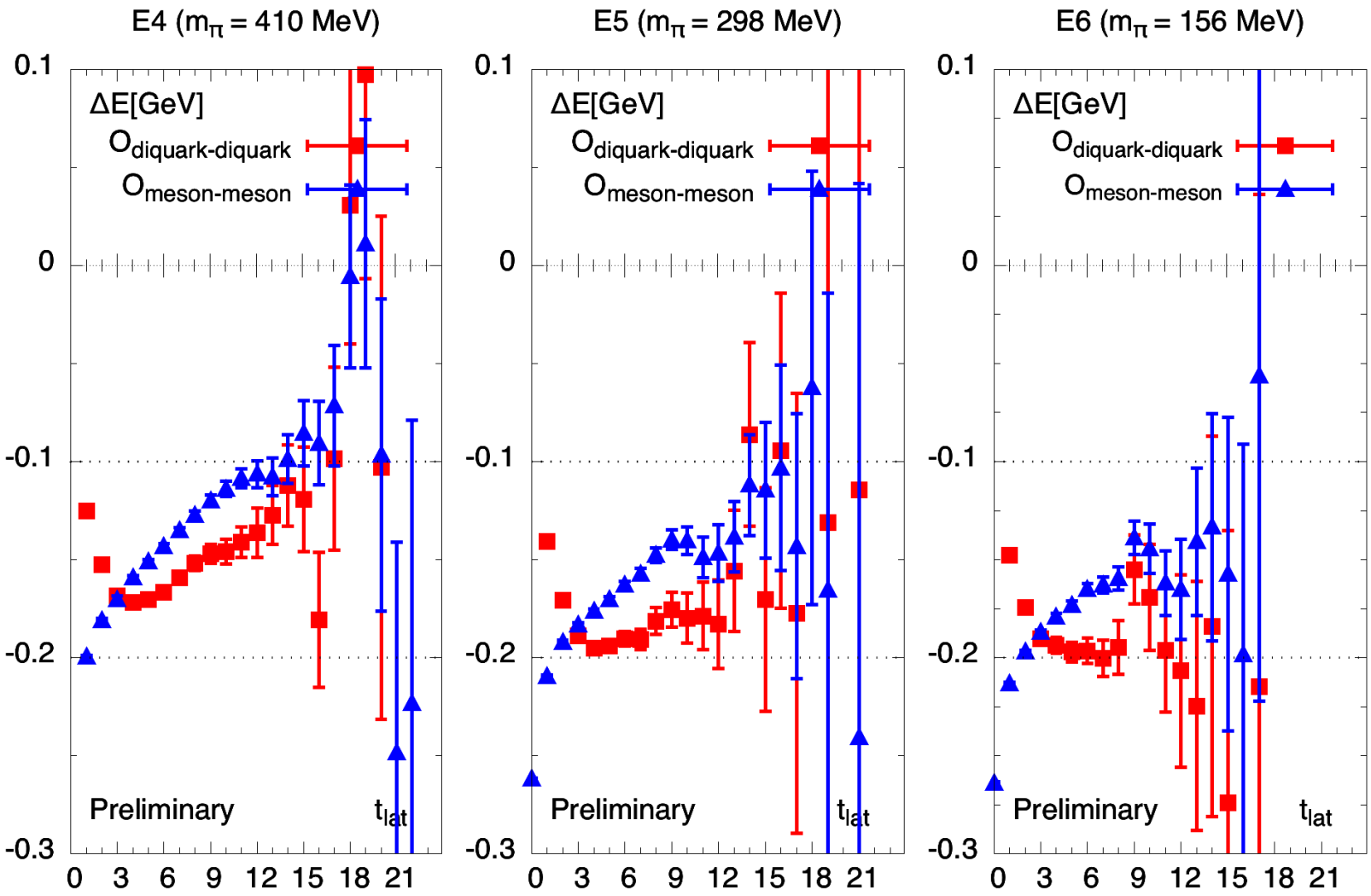


$$G_{\rm bind}(t) = \frac{C_{\rm tetra}(t)}{C_B(t)C_{B^*}(t)}$$

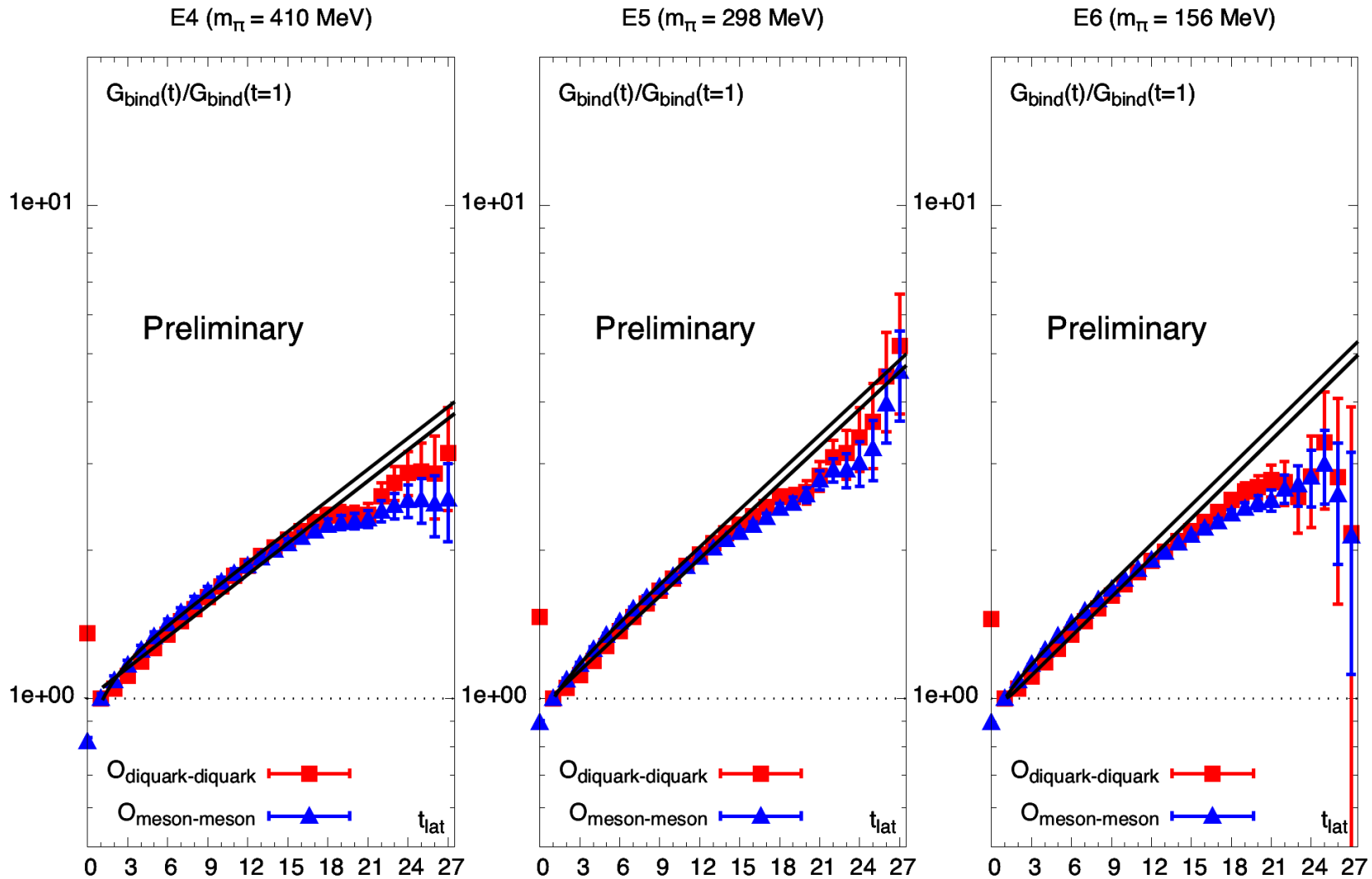
$ud\bar{b}\bar{b}$ tetraquark



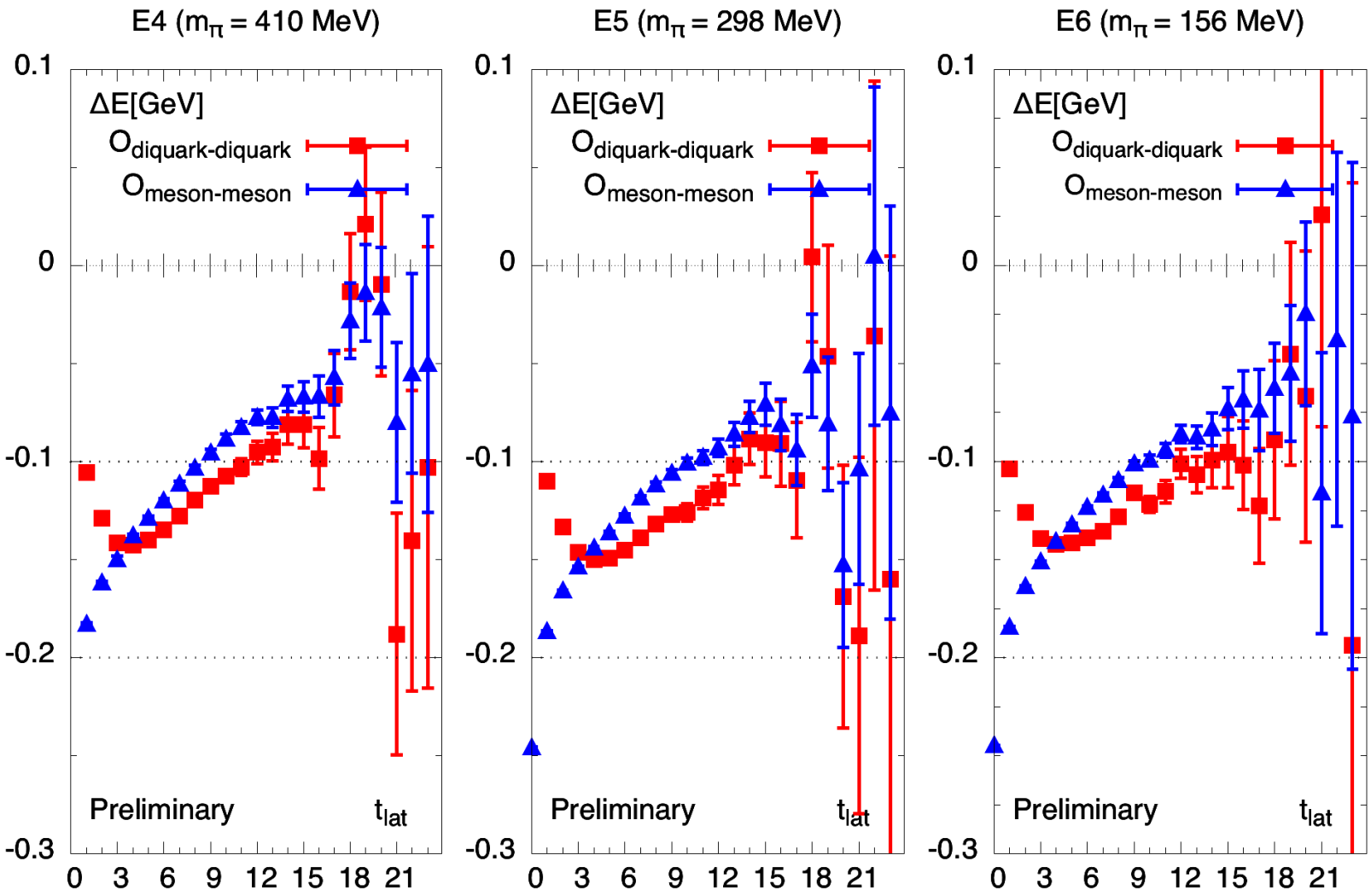
$ud\bar{b}\bar{b}$ tetraquark



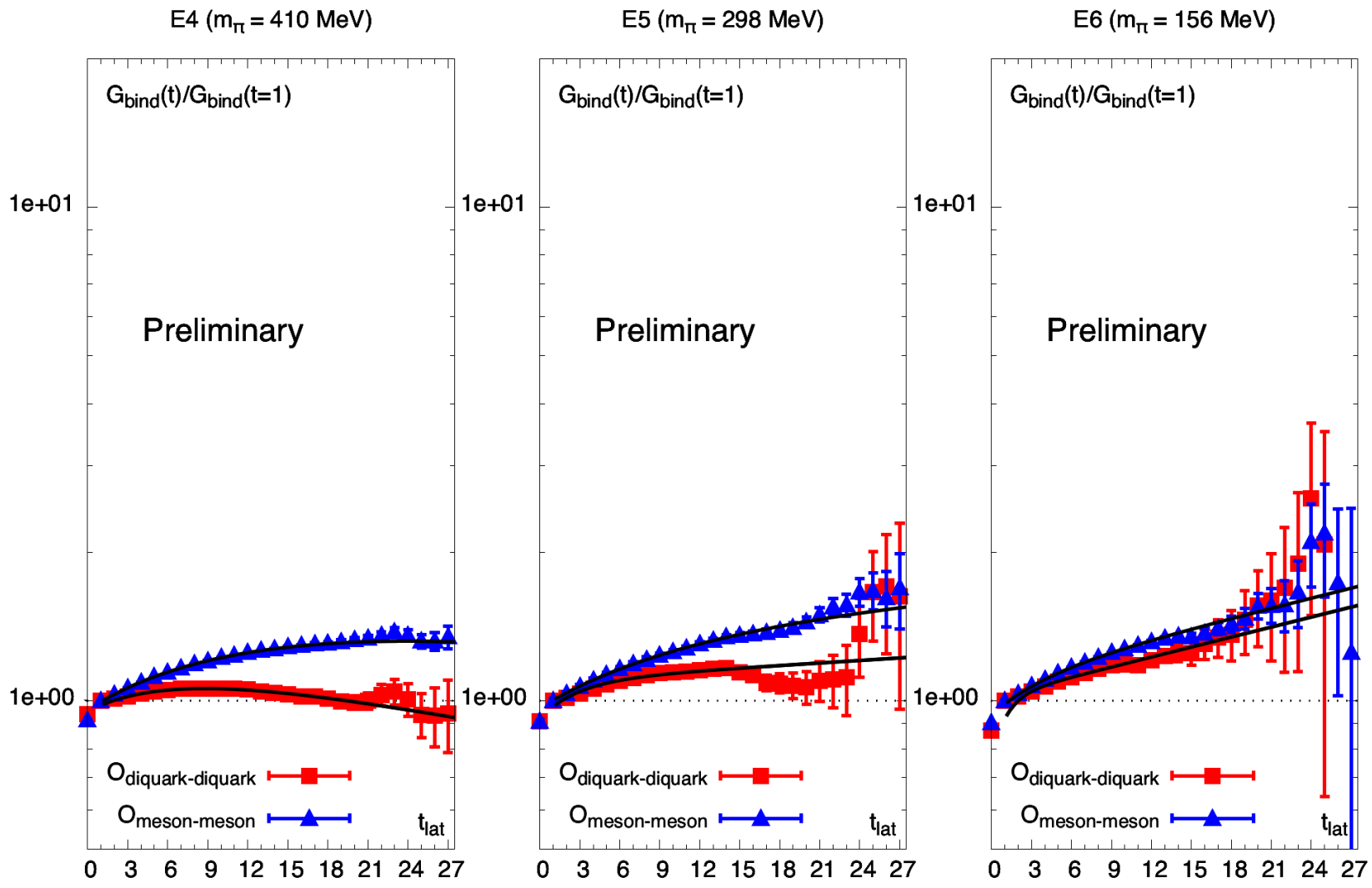
$\ell s \bar{b} \bar{b}$ tetraquark



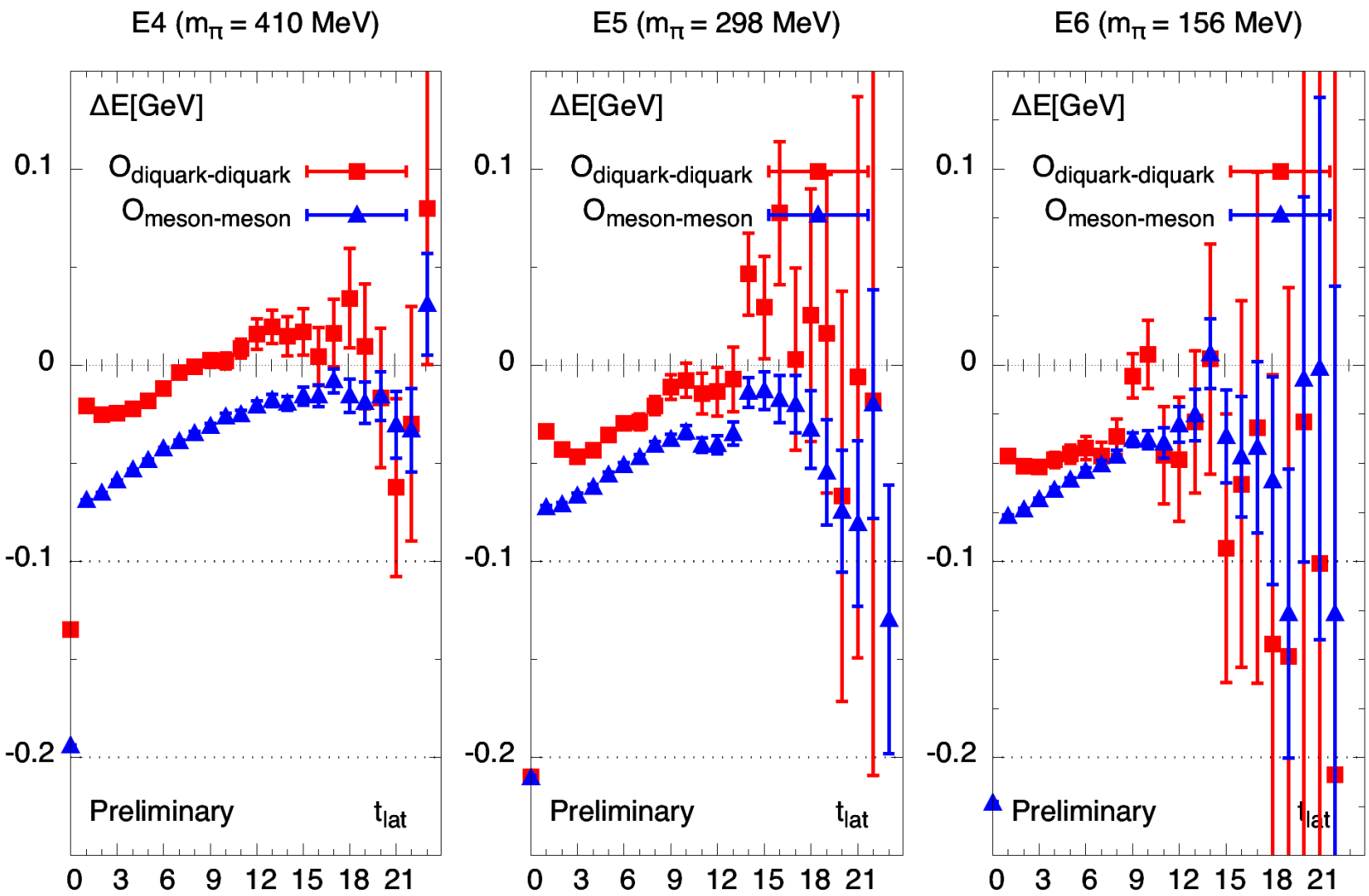
$\ell s \bar{b} \bar{b}$ tetraquark



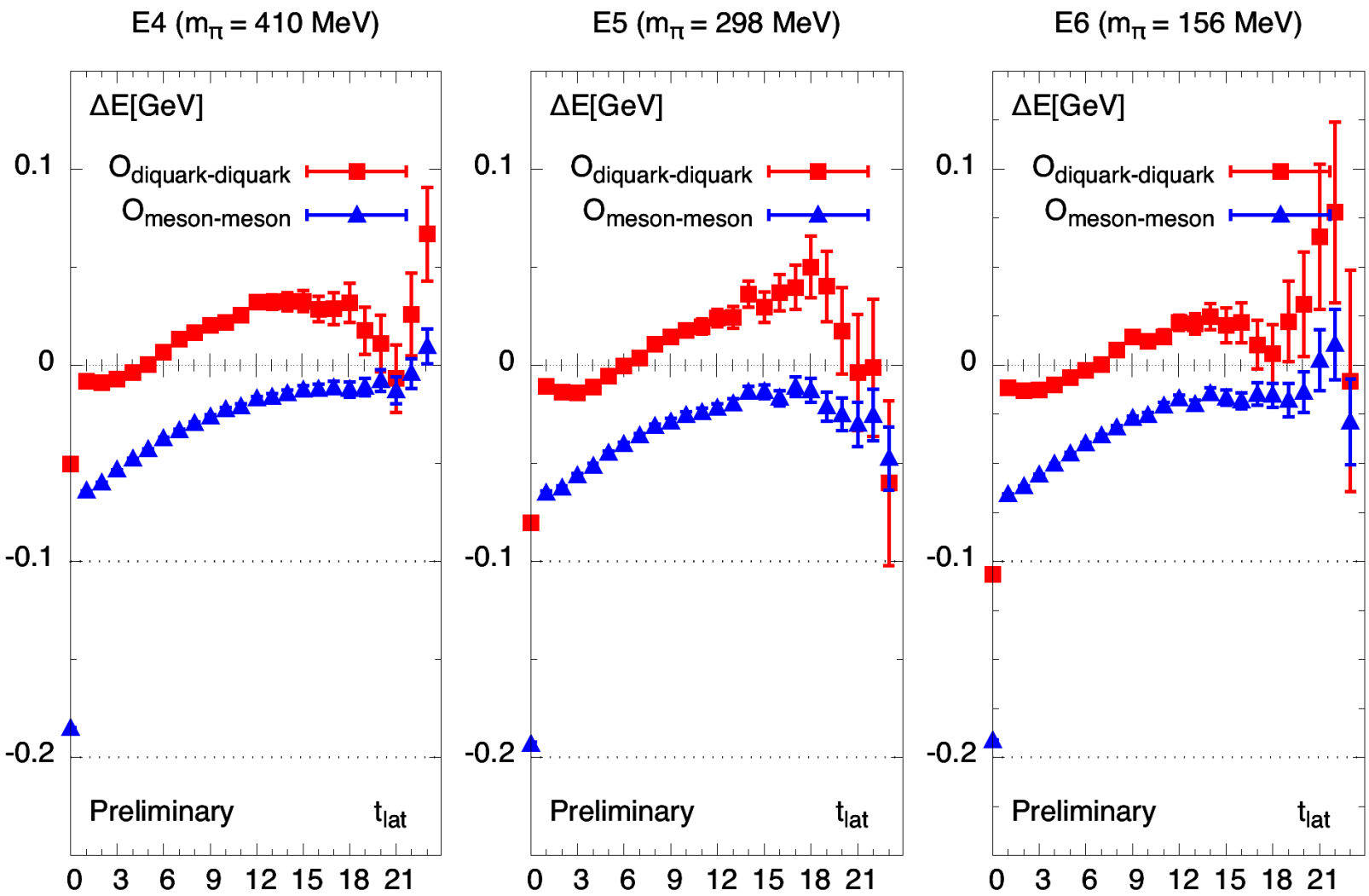
$ud\bar{c}\bar{c}$ tetraquark



$ud\bar{c}\bar{c}$ tetraquark



$\ell s \bar{c}\bar{c}$ tetraquark



summary

$J^P = 1^+$ tetraquark with flavor $qq\bar{Q}\bar{Q}$ is convenient for lattice:

- no disconnected diagrams.
- tetraquark appears as lightest state not as excited state.

This tetraquark has a favorable diquark structure for qq.
The $\bar{Q}\bar{Q}$ is related to known baryons by heavy quark symmetry.

Preliminary lattice results show deepest binding for $ud\bar{b}\bar{b}$.
Preliminary lattice results show evidence that $\ell s\bar{c}\bar{c}$ is unbound.

Expected decays: $ud\bar{b}\bar{b}$ will decay weakly.
$\ell s\bar{b}\bar{b}$ will decay weakly.
$ud\bar{c}\bar{c} \to DD\gamma$ is available (electromagnetic).