Lattice QCD investigation of BB and $B\bar{B}$ four-quark systems

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in collaboration with Pedro Bicudo, Krzysztof Cichy, Luka Leskovec, Stefan Meinel and Marc Wagner

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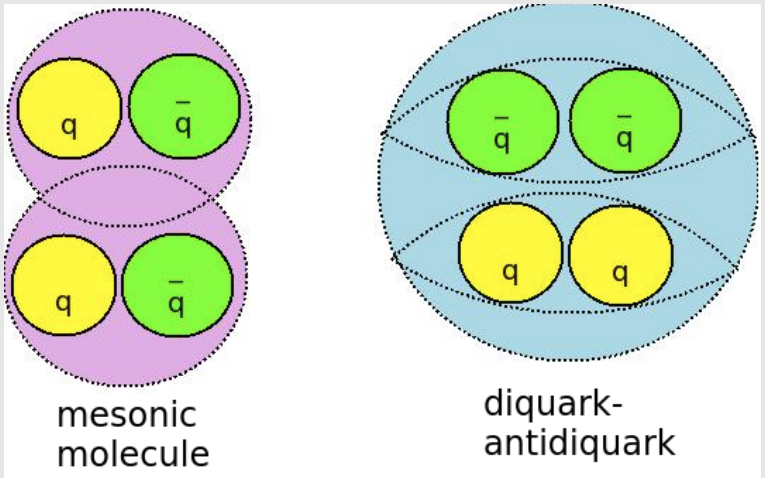
Study of four-quark states

Motivation

- A number of mesons observed in particle detectors (LHCb, Belle) is not well understood.
- E.g. charged charmonium- and bottomonium-like states ($Z_c{}^{\pm}$ and $Z_b{}^{\pm}$)
- They include $b\bar{b}$ or $c\bar{c}$, but are also charged: must be 4-quark states



possible tetraquark structures

The static-light approach

- Computation of 4-quark states very difficult
- If 2 quarks are heavy and 2 quarks are light: Treat degrees of freedom independently in two steps (Born-Oppenheimer approximation [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Ann.Phys. 389, Nr. 20, 1927]).

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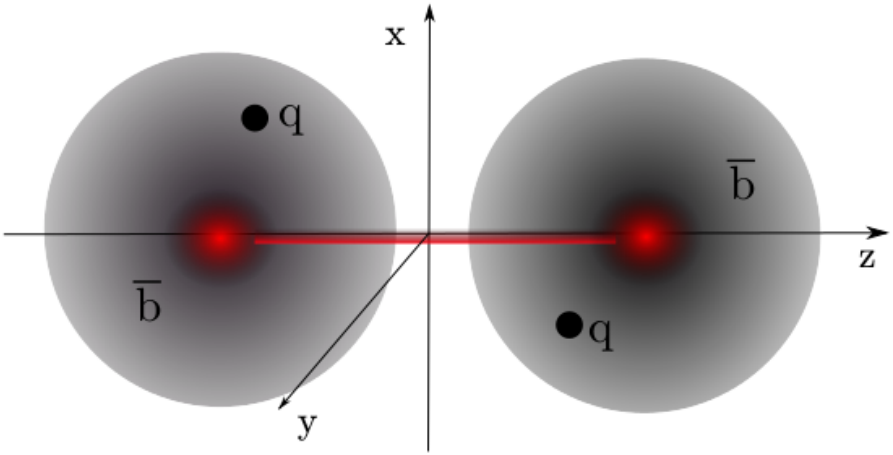
  1. Lattice computation of the potential of two static quarks in the presence of two light quarks, i.e. potential can be interpreted as the potential between two B mesons



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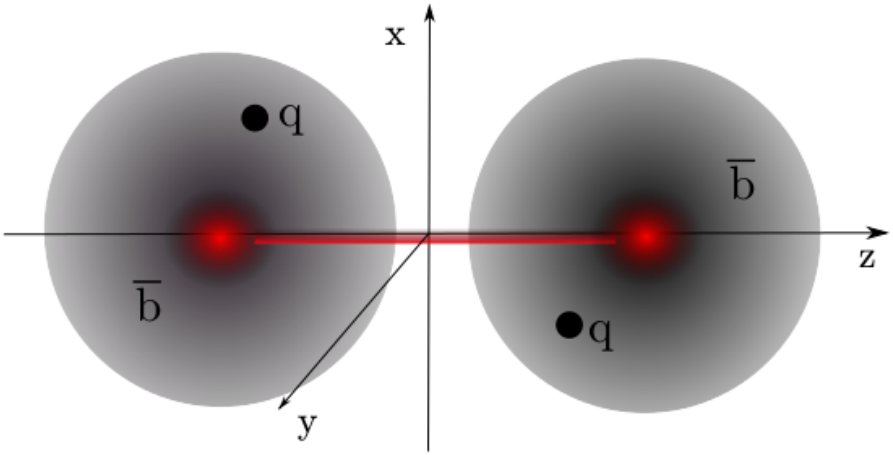

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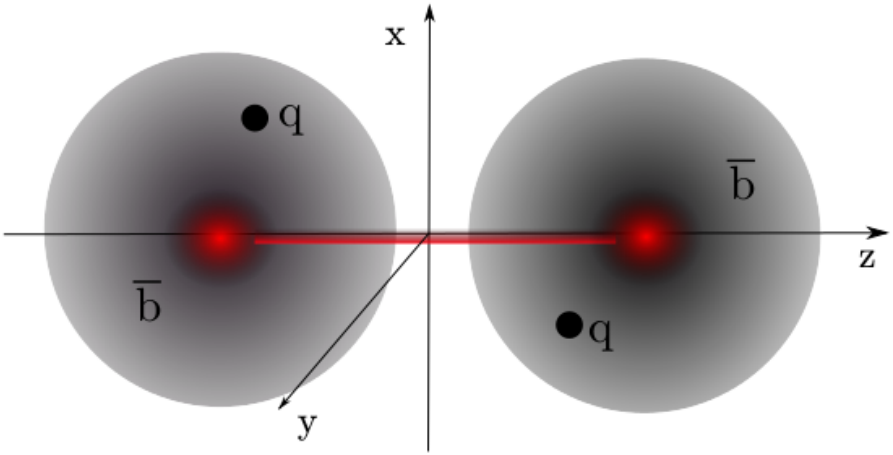

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Hadron spectroscopy II

- Obtain the correlation function in time for each separation $r = |\vec{x} - \vec{y}|$ of the static quarks via

$$C(t) = \langle \Omega | \, \mathcal{O}^\dagger(t) \mathcal{O}(0) \, | \Omega \rangle$$

Requires $\mathcal{O}(\mathrm{months})$ of computing time on high performance computers.

- For large t one finds the potential $V(r)$ of the hadronic state $\mathcal{O}(t)$:

$$\lim_{t \to \infty} \langle \Omega | \, \mathcal{O}^\dagger(t) \mathcal{O}(0) \, | \Omega \rangle \propto \exp(-V(r)t)$$

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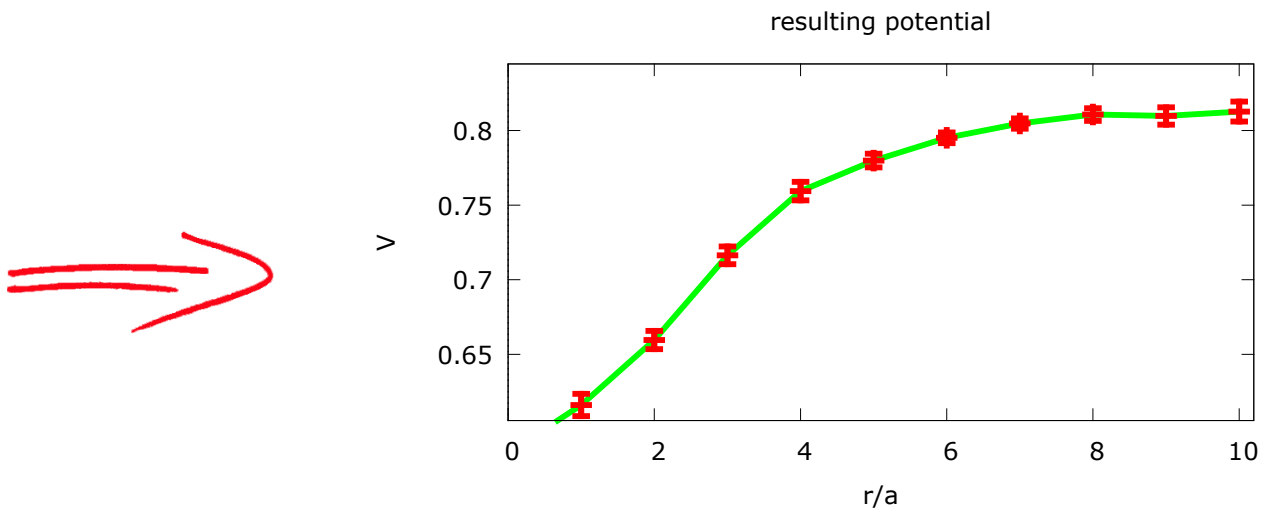

BB systems in the static-light approach

P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, Phys. Rev. D **92** (2015) no.1, 014507 doi:10.1103/PhysRevD.92.014507 [arXiv:1505.00613 [hep-lat]].

P. Bicudo, K. Cichy, A. Peters and M. Wagner, Phys. Rev. D **93** (2016) no.3, 034501 doi:10.1103/PhysRevD.93.034501 [arXiv:1510.03441 [hep-lat]].

related to previous papers:

C. Michael *et al.* [UKQCD Collaboration], Phys. Rev. D **60** (1999) 054012 doi:10.1103/PhysRevD.60.054012 [hep-lat/9901007].

M. S. Cook and H. R. Fiebig, hep-lat/0210054.

T. Doi, T. T. Takahashi and H. Suganuma, AIP Conf. Proc. **842** (2006) 246 doi:10.1063/1.2220239 [hep-lat/0601008].

W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76** (2007) 114503 doi:10.1103/PhysRevD.76.114503 [hep-lat/0703009 [HEP-LAT]].

M. Wagner [ETM and Y Collaborations], PoS LATTICE **2010** (2010) 162 [arXiv:1008.1538 [hep-lat]].

G. Bali *et al.* [QCDSF Collaboration], PoS LATTICE **2010** (2010) 142 [arXiv:1011.0571 [hep-lat]].

The BB four-quark operator

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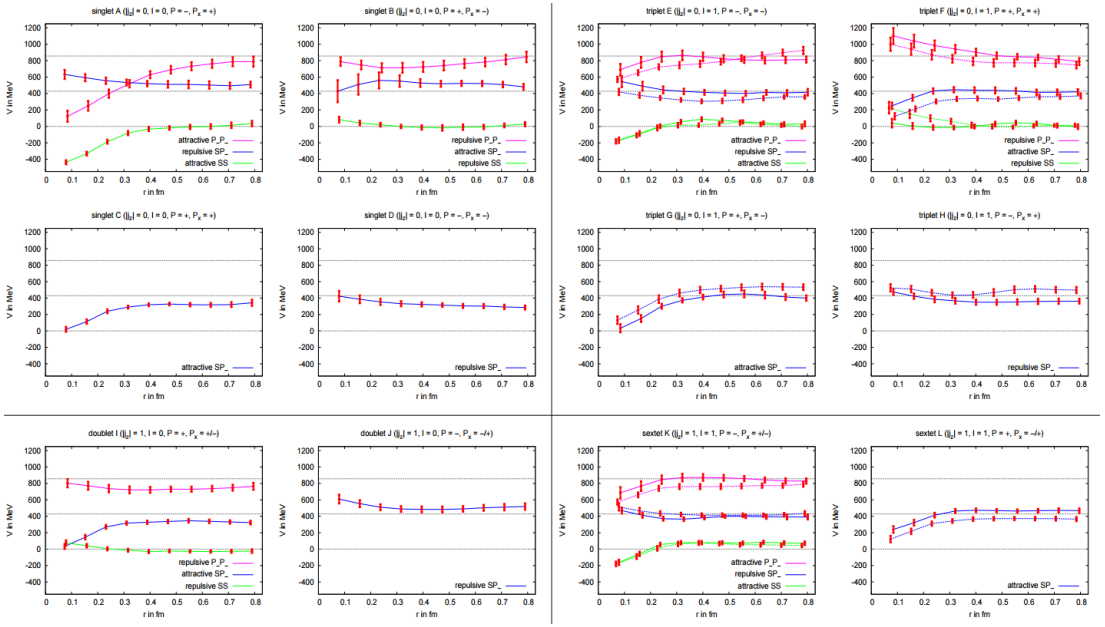

2 attractive ground state channels:

- spin scalar isosinglet with $I(J^P) = 0(1^+)$
- spin vector isotriplet $I(J^P) \in \{0(1^+), 1(1^+), 1(2^+)\}$

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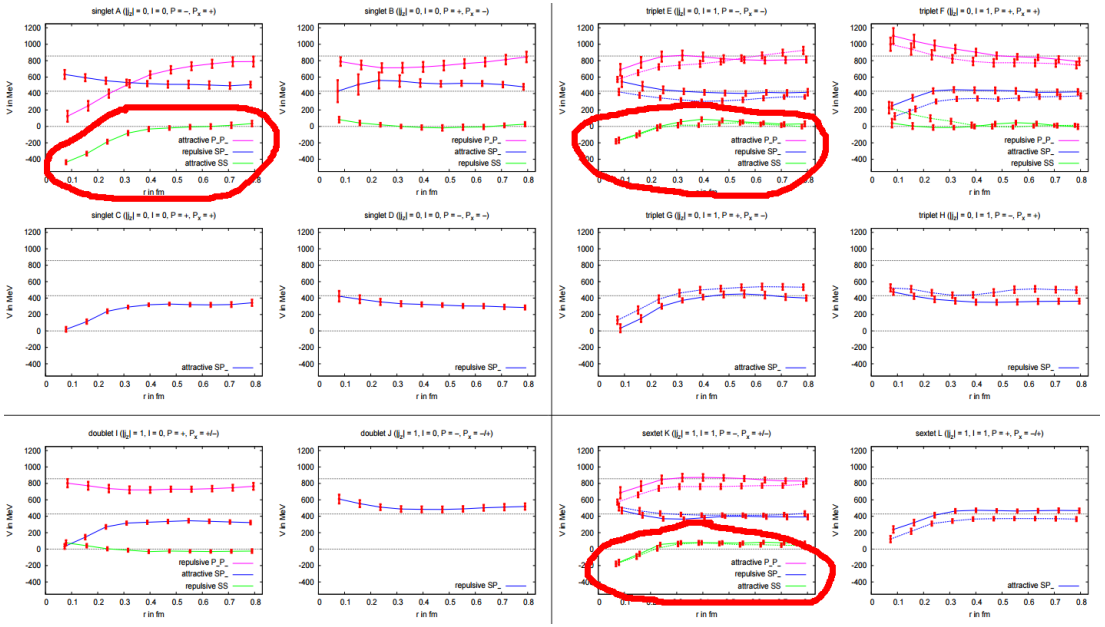

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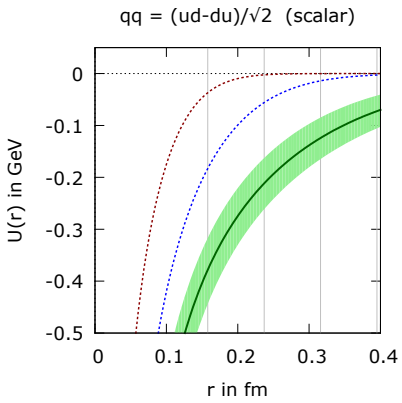

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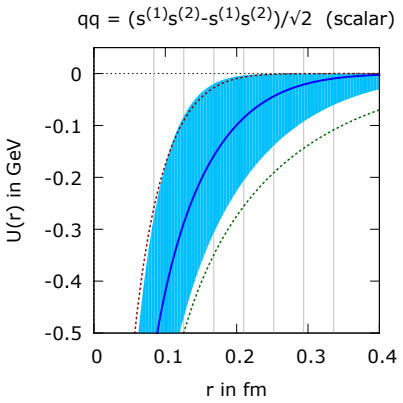

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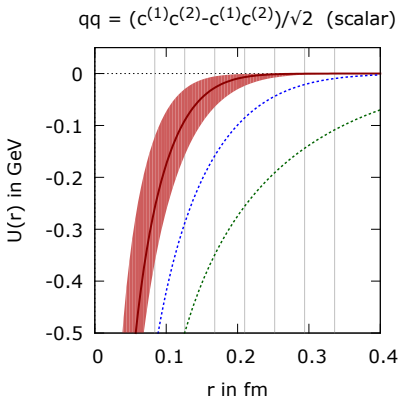

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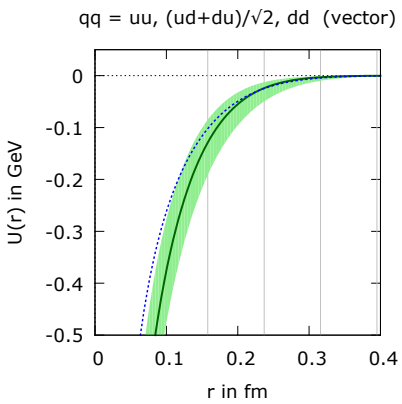
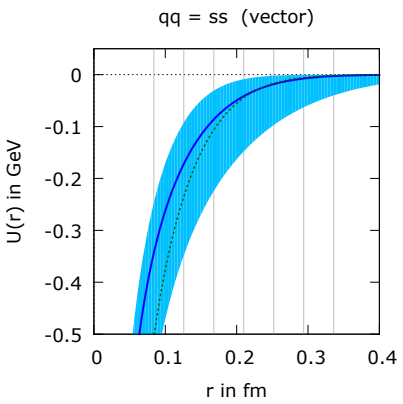
vector isotriplet





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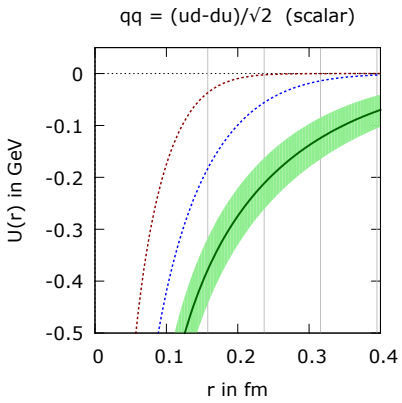

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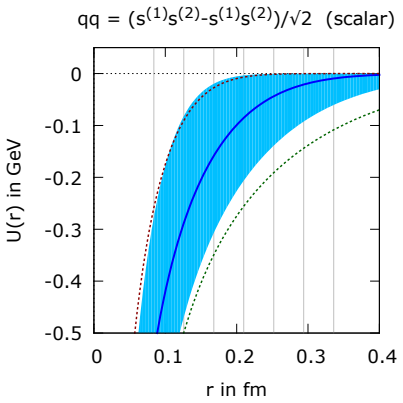

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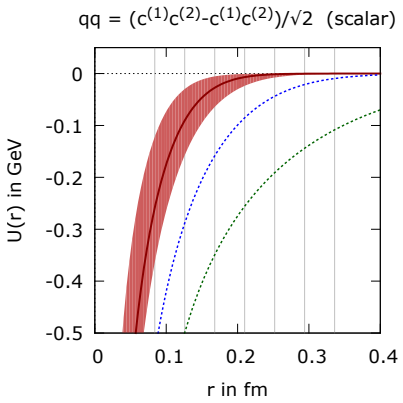

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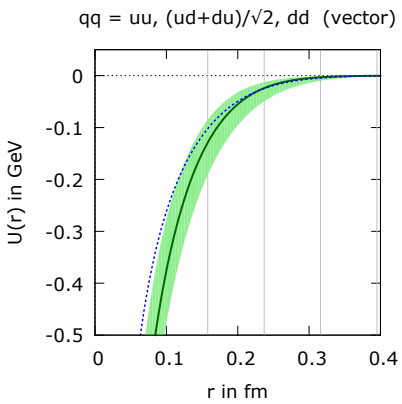

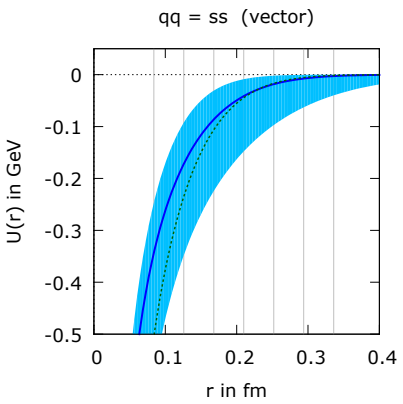

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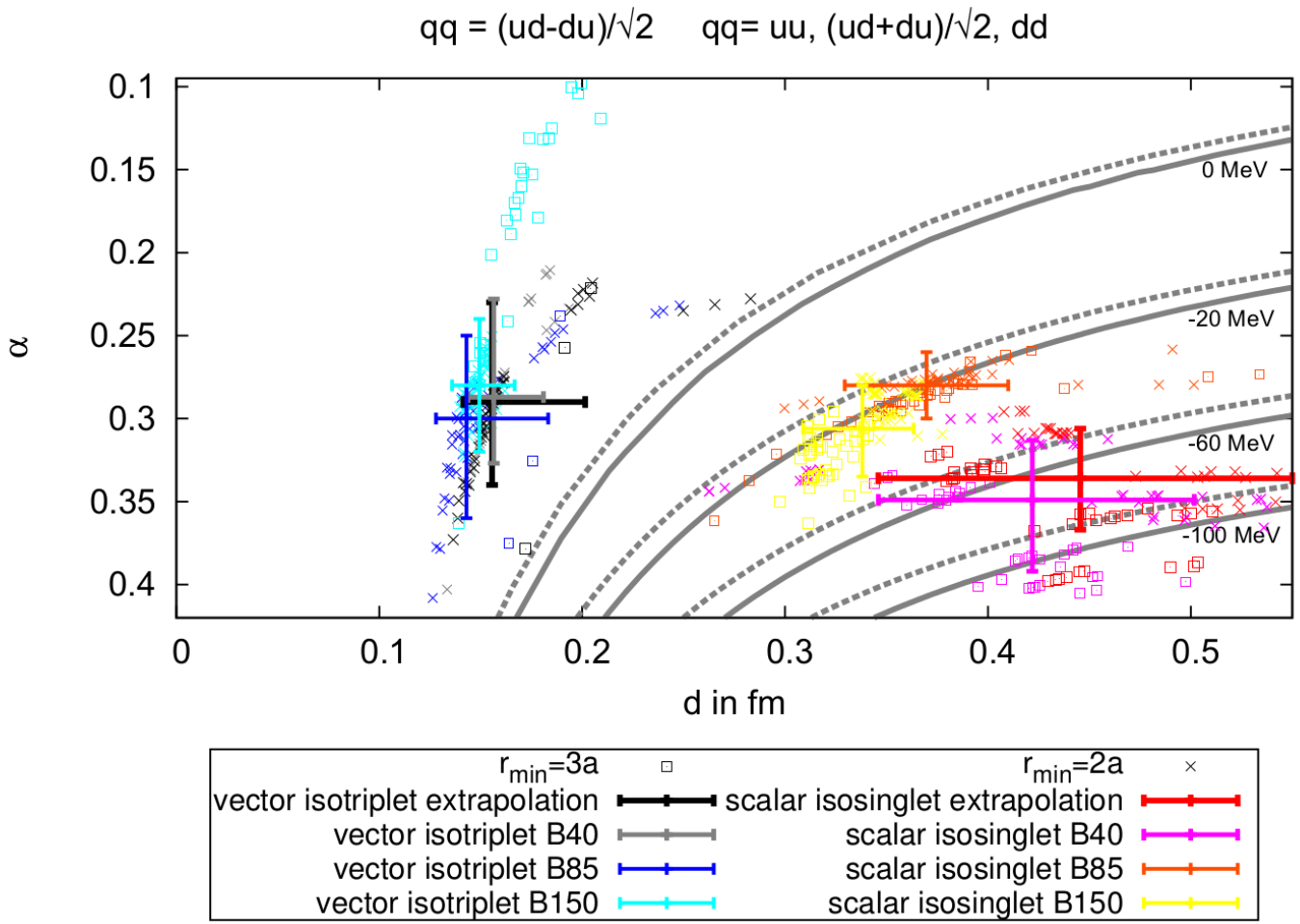
Most promising channel for a bound state: scalar $ud\bar{b}\bar{b}$

remember:

$$V(r) = -\frac{\alpha}{r}\,\mathrm{e}^{-\left(\frac{r}{d}\right)^2}$$

Extrapolation to the physical quark mass: Binding increases



For the BB four-quark state with quantum numbers $I(J^P) = 0(1^+)$ we find $E_B = -90^{+43}_{-36}$ MeV at the physical point
$\longrightarrow$ binding with more than 2σ confidence level

Study of BB^* systems by means of NRQCD

in collaboration with

Stefan Meinel and Luka Leskovec

Work in progress.

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD

- strong evidence for a $ud\bar{b}\bar{b}$ bound state in the $I(J^P) = 0(1^+)$ channel from static-light approach
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- $n_f = 2 + 1$ dynamical sea quarks, $m_\pi = 336$ MeV [arXiv:1409.0497]

The NRQCD BB^* correlator:

$$\sum_{\vec{x}_1,\vec{x}_2,\vec{x}_1',\vec{x}_2'} \mathrm{e}^{i\vec{p}_1(\vec{x}_1-\vec{x}_1')}\mathrm{e}^{i\vec{p}_2(\vec{x}_2-\vec{x}_2')}\delta_{\vec{x}_1,\vec{x}_2}\delta_{\vec{x}_1',\vec{x}_2'}$$
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The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD - preliminary results

- To get a clear signal in a first step we use unphysically heavy $\bar{b}$ quarks
- compute masses of B, B^* and BB^*
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Qualitative confirmation of static-light result

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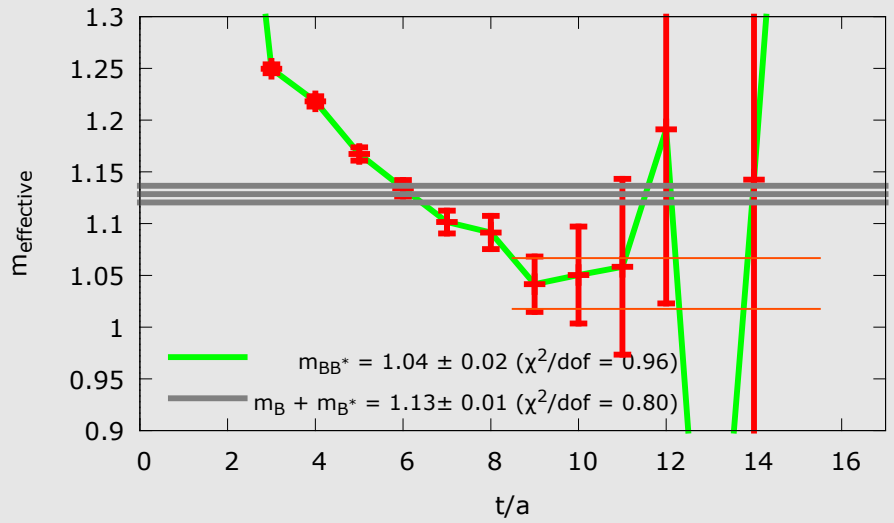

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$\Rightarrow$ Qualitative confirmation of static-light result

$B\bar{B}$ systems in the static-light approach

A. Peters, P. Bicudo, K. Cichy and M. Wagner, "Investigation of $B\bar{B}$ four-quark systems using lattice QCD," arXiv:1602.07621 [hep-lat].

Work in progress.

The $B\bar{B}$ system - Additional problem

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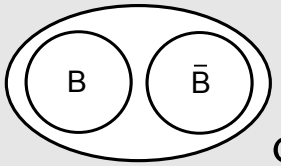
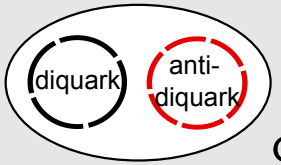
  a mesonic molecule 

  or

  a diquark-antidiquark 

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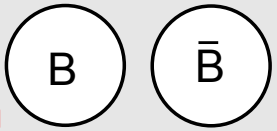
  a B meson and a far separated $\bar{B}$ meson 

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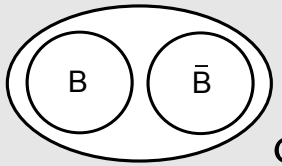
  a mesonic molecule 

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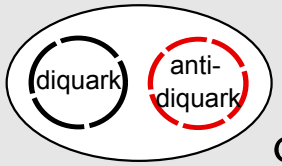
  a diquark-antidiquark 

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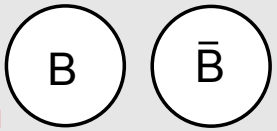
  a B meson and a far separated $\bar{B}$ meson 

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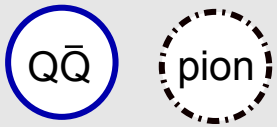
  $Q\bar{Q} + \pi$ 

To separate these structures, we ...

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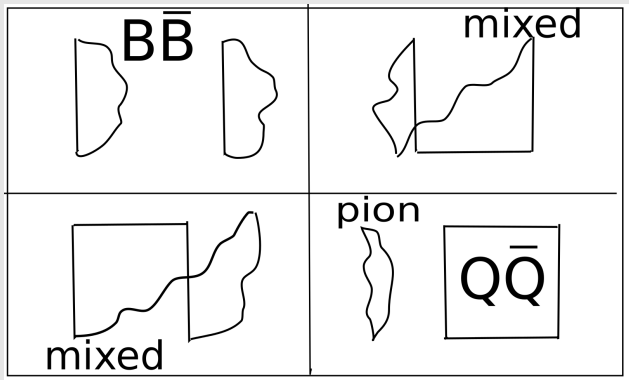

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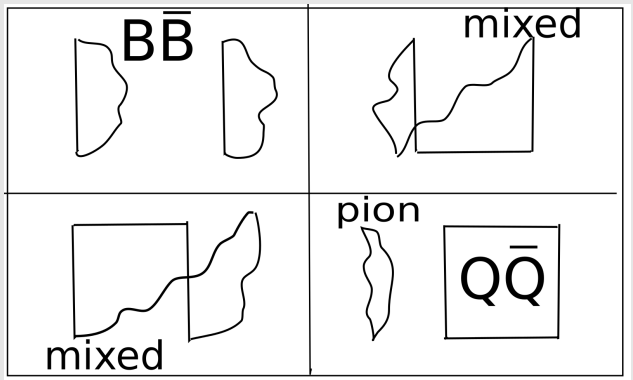

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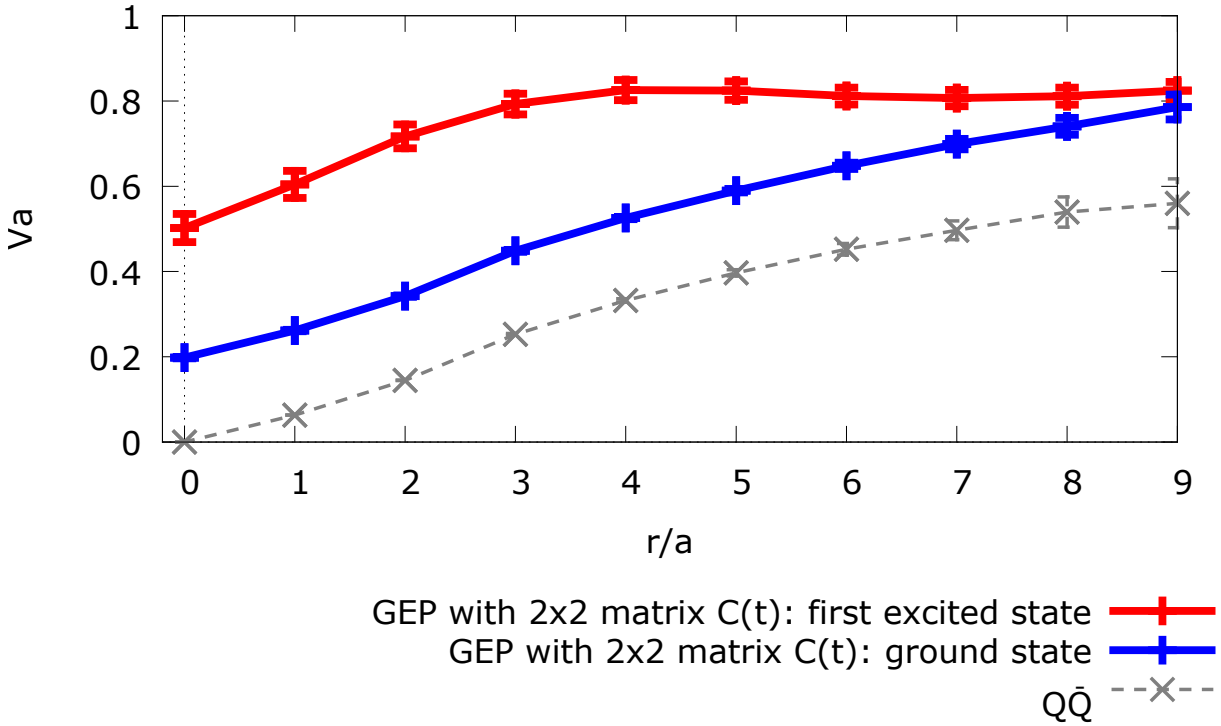

Potentials obtained

- $Q\bar{Q} + \pi$: ground state (blue)
- first excited state of the 2x2 matrix: free of contributions of $Q\bar{Q} + \pi$ (red)

Preliminary results II

Binding energy

- Identical analysis as in the BB case (Schrödinger's equation) yields for quantum numbers $I(J^P) = 1(1^+)$ (i.e. Z_b^+):

$$E_B = (-58 \pm 71)\mathrm{MeV}$$

- very vague indication for a $\bar{u}d\bar{b}b$ bound state

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Backup

Meson content of a four-quark state

- One can extend the meson content of the four-quark states by application of the following light quark projectors on $\mathcal{O}$:
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small separations of the static antiquarks:

- interaction due to 1-gluon exchange
- bound state: static $\bar{Q}\bar{Q}$ pair in a color triplet (attractive) $\longrightarrow$ antidiquark

large separations of the static antiquarks:

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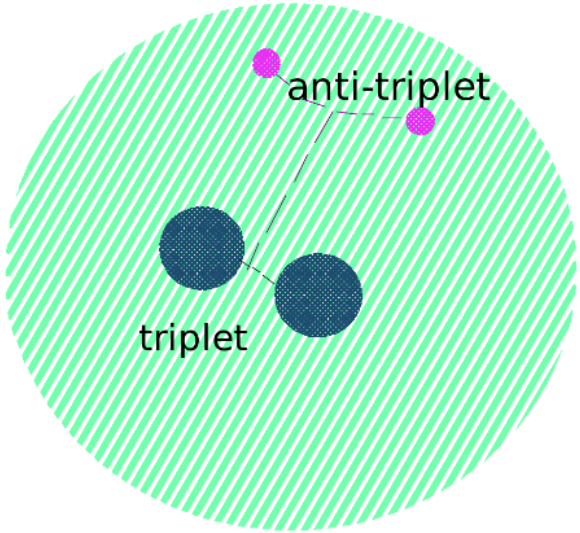
- basically 2 static-light mesons



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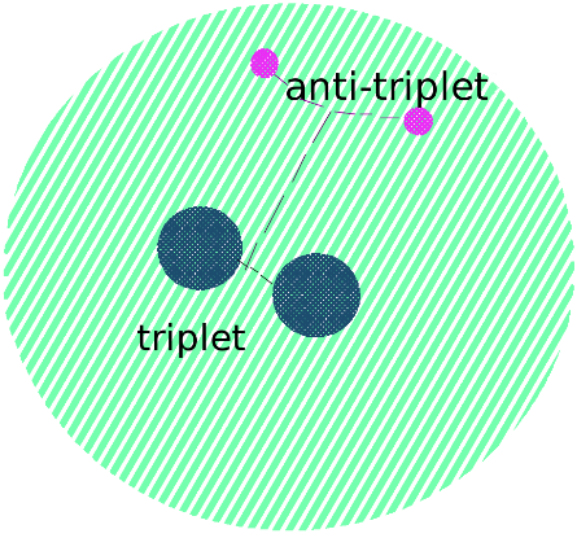

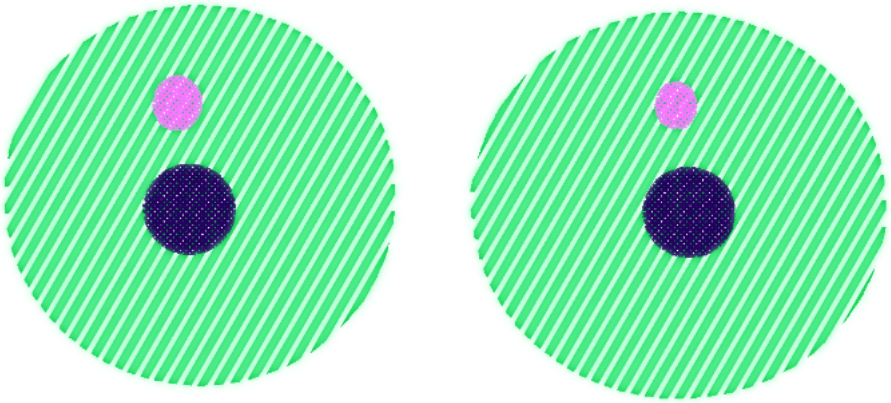

Lattice Setups

BB and $B\bar{B}$ in the static-light approach:

Ens.	β	lattice	$a\mu$	m_π[MeV]	a [fm]	L [fm]	confs
B40.24	3.90	$24^3 \times 48$	0.0040	340	0.0790(26)	1.9	480
B85.24	3.90	$24^3 \times 48$	0.0085	480	0.0790(26)	1.9	400
B150.24	3.90	$24^3 \times 48$	0.0150	650	0.0790(26)	1.9	260

Gauge configurations generated by ETMC

BB^* with NRQCD:

Ens.	β	lattice	$am_{u,d}$	am_s	m_π[MeV]	a [fm]	L [fm]	confs
C54	2.13	$24^3 \times 64$	0.005	0.04	336	0.1119(17)	2.7	1676

Gauge configurations generated by RBC and UKQCD collaborations

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Perform a large number of fits varying...

- the temporal separation at which the lattice potential is read of the correlation function

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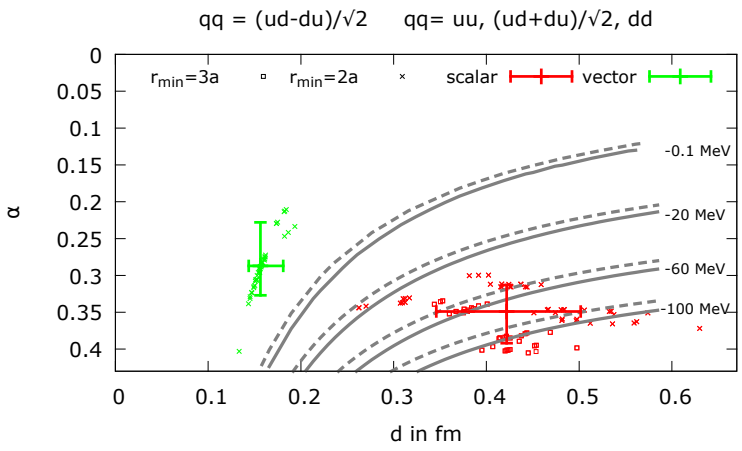

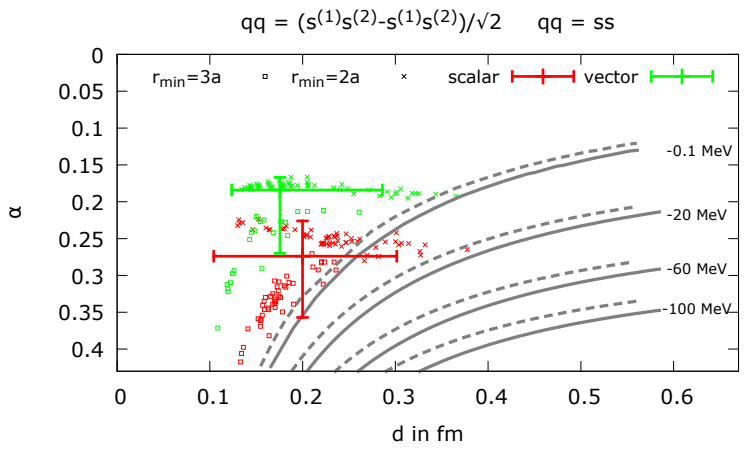

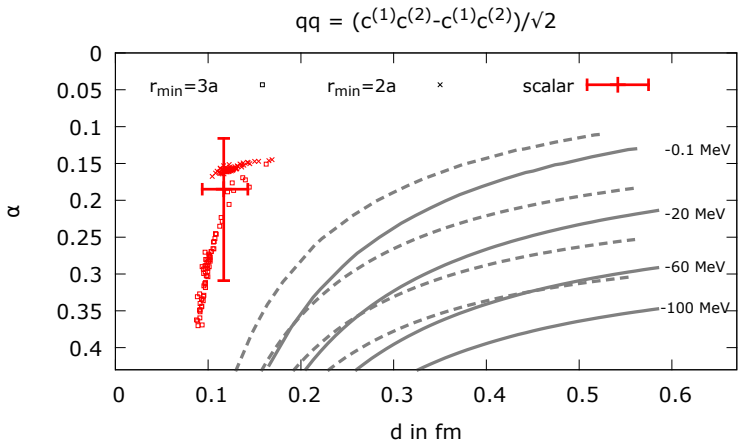

Binding for light isosinglet channel only!

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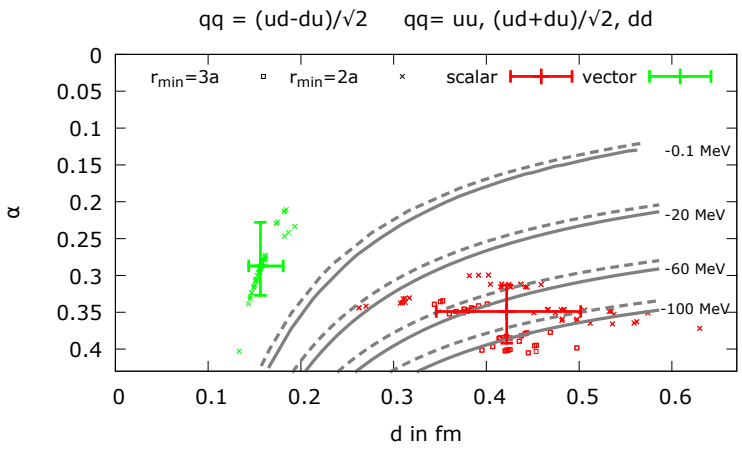
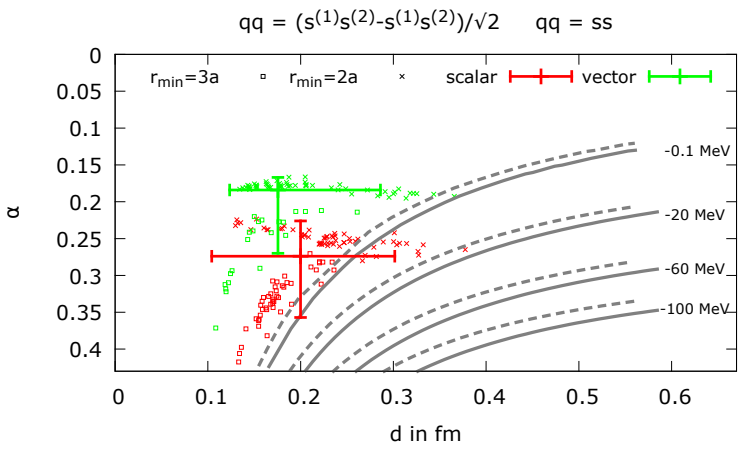
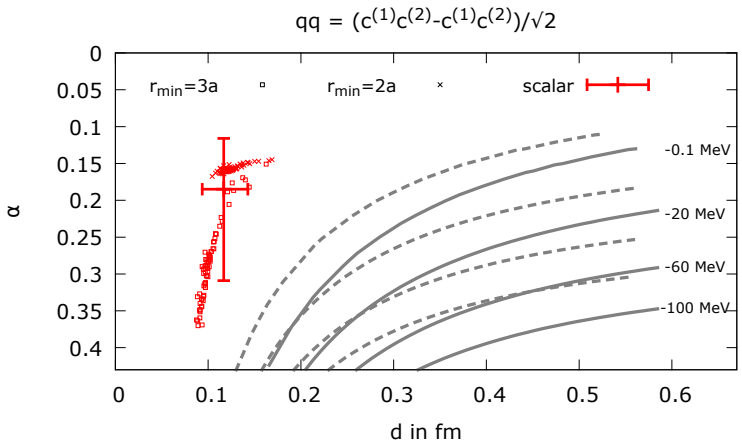


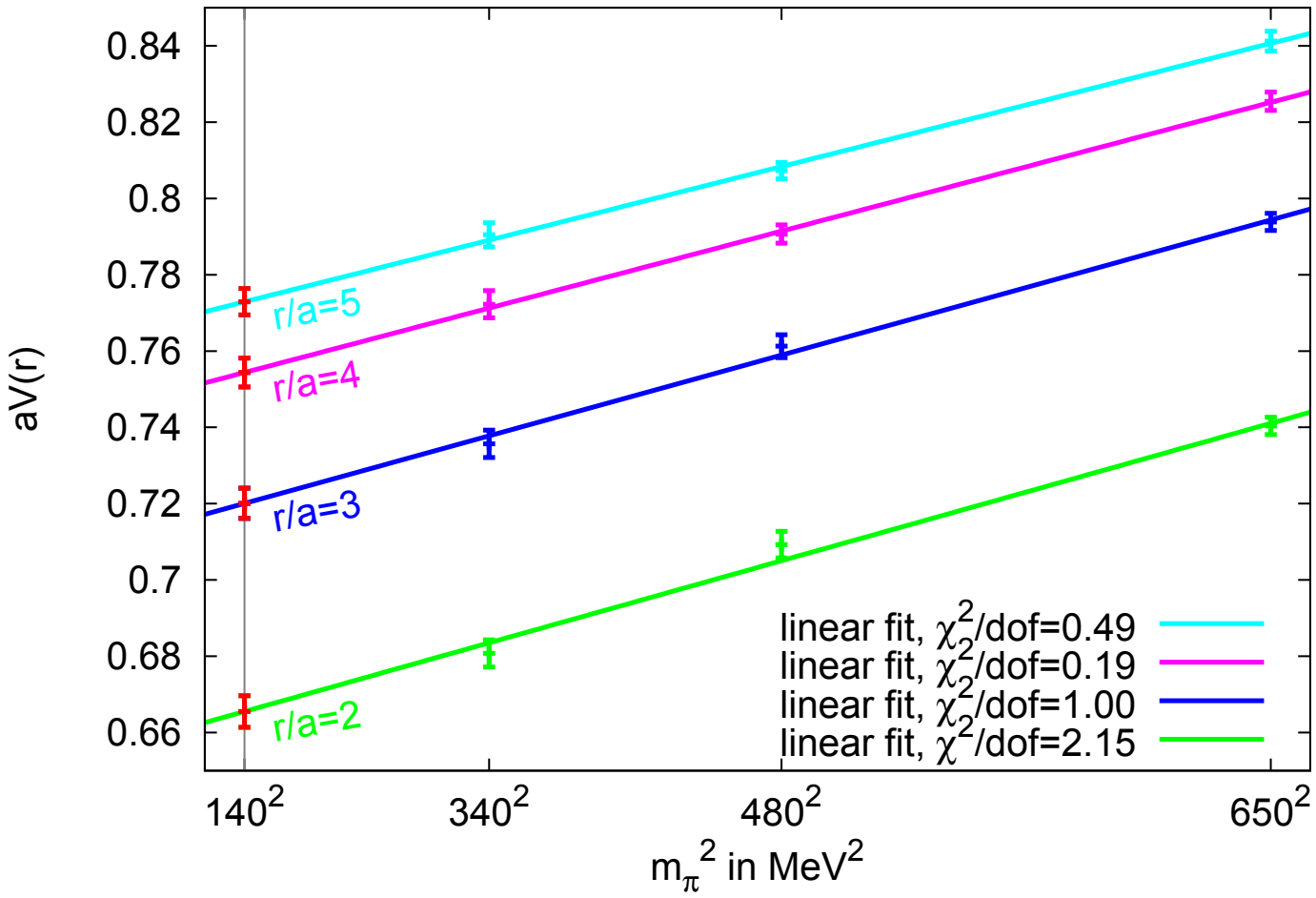




Binding for light isosinglet channel only!

Extrapolation to the physical pion mass

Example plots for a t-range $[4a...9a]$



The GEP

1. Build a matrix $C(t)$ of correlation functions $C_{ij}(t)$
2. Solve the GEP:

$$C_{jk}(t)v_k^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)C_{jk}(t_0)v_k^{(n)}(t,t_0)$$

3. And find:

$$m_{\mathrm{eff}}^{(n)}(t,t_0) = \lim_{t\to\infty} \frac{1}{a} \log \frac{\lambda^{(n)}(t,t_0)}{\lambda^{(n)}(t+a,t_0)}$$

Choice of Γ for the $B\bar{B}$ system

- The choice of the matrix Γ is constrained by the quantum numbers of the $\mathcal{O}_{Q\bar{Q}+\pi}$
- Only taking into account $\mathcal{O}_{B\bar{B}}$ we find the strongest attraction for $\Gamma = \gamma_5 - \gamma_0\gamma_5$