Practical Parametrization for Line Shapes of Near-Threshold States

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Setting the stage

Many of the new, unexplained states located near thresholds, sometimes overlapping with other structures

Example: $Z_b(10610)$ and $Z_b(10650)$

Goals

$\rightarrow$ Analyze data with theoretically sound tools in order to

$\rightarrow$ extract properties of states in the least model dependent way

$\rightarrow$ from an easy to use parametrization

$\rightarrow$ in order to understand the inner workings of QCD;

Key references:

C. H. et al. Phys. Rev. Lett. **115** (2015) no.20, 202001 [arXiv:1507.00382 [hep-ph]].

F.-K. Guo et al., Phys. Rev. D **93** (2016) no.7, 074031 [arXiv:1602.00940 [hep-ph]].

What is a state?

The S–matrix is characterized by its analytic structure (in s).

→ branch points (and the corresponding cuts)

⟶ at each channel opening for $s > s^{\rm thres}$: right hand cut

⟶ in the crossed channels for $s < s^{\rm thres}$: left hand cut

⟶ inside the un-physical sheet (see below)

→ poles on the physical sheet: bound states

⟶ only for real $s < s^{\rm thres}_{\rm min}$ (no other singul. allowed here)

→ poles on the un-physical sheet (closest to the physical one)

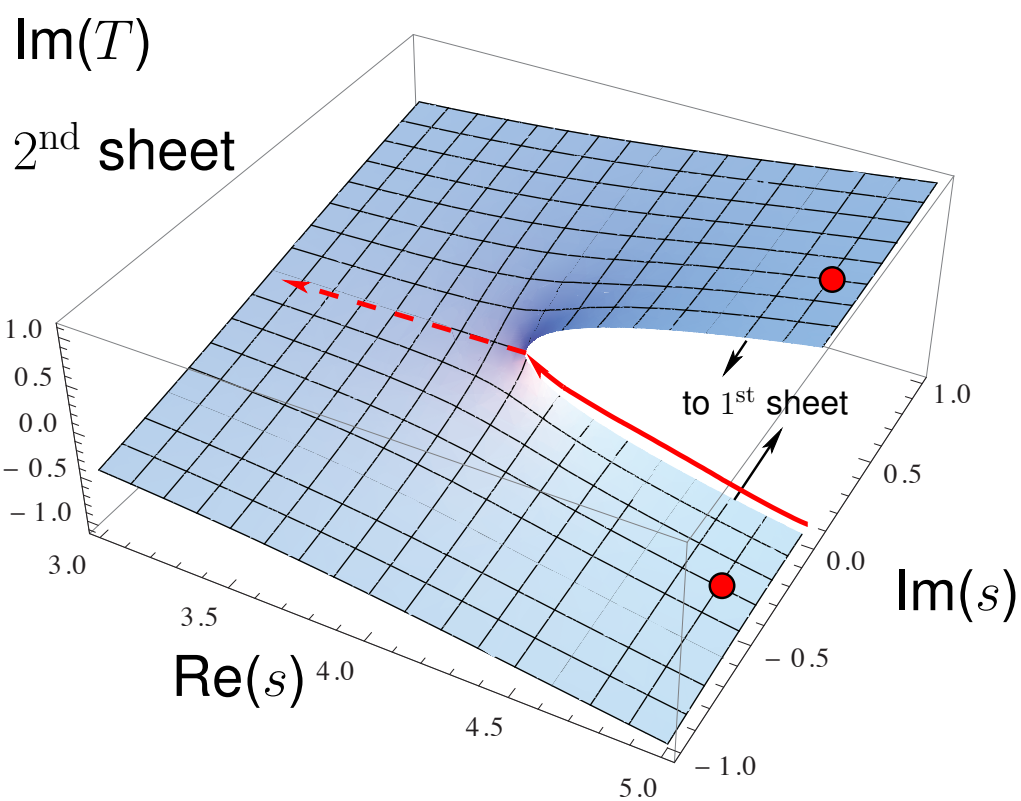
⟶ for real $s < s^{\rm thres}_{\rm min}$: virtual state

⟶ for complex s: resonance

Why not to use BWs I: Analyticity

$\rightarrow$ For real $s < s_{\rm min}^{\rm thres}$, S is real $\rightarrow$ Branchpoint at $s = s^{\rm thres}$

$\rightarrow$ $S(s^*) = S^*(s) \longrightarrow$ pole at s implies pole at s^*



For narrow resonances:

In resonance region:
only lower pole matters

At threshold:
both poles important!

For broad resonances:

always both important

Keep track of the cuts!

Why not to use BWs II: Unitarity

For one channel only one has: $\mathrm{Im}(T) = \sigma |T|^2$

where $\sigma = \sqrt{1 - 4m^2/s}$ is the phase space. Then for

$$T = -\frac{\mathrm{res}_{(1)}{}^2}{s - M_1^2 + iM_1\Gamma_1} - \frac{\mathrm{res}_{(2)}{}^2}{s - M_2^2 + iM_1\Gamma_2}$$

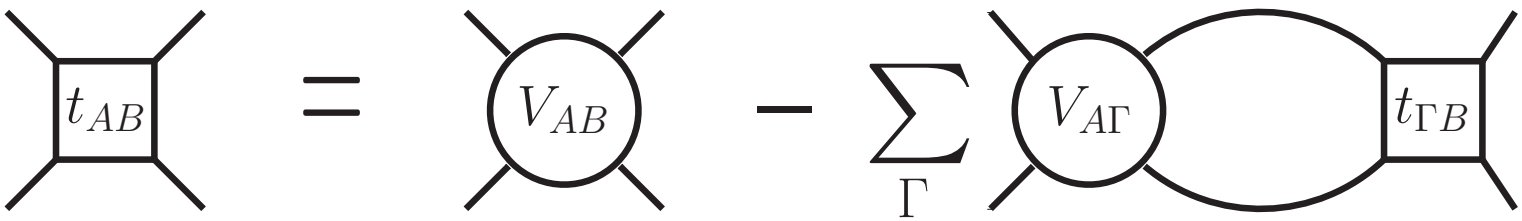
we get using $\sigma \left|\mathrm{res}_{(\mathrm{i})}\right|^2 = M_i\Gamma_i(s)$ (for $\mathrm{res}_{(i)}$ real)

$$\begin{aligned}
\mathrm{Im}(T) &= \frac{\mathrm{res}_{(1)}{}^2\Gamma_1\mathrm{M}_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\mathrm{res}_{(2)}{}^2\Gamma_2\mathrm{M}_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2} \\
\sigma|T|^2 &= \frac{\mathrm{res}_{(1)}{}^2\Gamma_1\mathrm{M}_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\mathrm{res}_{(2)}{}^2\Gamma_2\mathrm{M}_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2} \\
&\quad + 2\sigma\mathrm{Re}\left(\frac{\mathrm{res}_{(1)}{}^2}{s - M_1^2 + iM_1\Gamma_1}\frac{\mathrm{res}_{(2)}{}^2}{s - M_2^2 - iM_1\Gamma_2}\right)
\end{aligned}$$

Do not sum Breit-Wigners — Interference term violates unitarity!

How to do better ...

Start from dynamical equation: Lippmann–Schwinger Equation



with $\quad b = \overline{1, N_{\mathsf{p}}} \quad \beta = \overline{1, N_{\mathsf{e}}} \quad i = \overline{1, N_{\mathsf{in}}}$

$$\hat{V} = \begin{pmatrix} v_{ab} & v_{a\beta}(\boldsymbol{p}') & v_{ai}(\boldsymbol{k}) \\ v_{\alpha b}(\boldsymbol{p}) & v_{\alpha\beta}(\boldsymbol{p}, \boldsymbol{p}') & v_{\alpha i}(\boldsymbol{p}, \boldsymbol{k}) \\ v_{jb}(\boldsymbol{k}') & v_{j\beta}(\boldsymbol{k}', \boldsymbol{p}') & v_{ji}(\boldsymbol{k}', \boldsymbol{k}) \end{pmatrix} \qquad \begin{array}{l} a = \overline{1, N_{\mathsf{p}}} \\ \alpha = \overline{1, N_{\mathsf{e}}} \\ j = \overline{1, N_{\mathsf{in}}}. \end{array}$$

$N_{\mathrm{p}} =$ # pole terms — included, if necessary

$N_{\mathrm{e}} =$ # elastic channels — here: 2 ($B^*\bar{B}, B^*\bar{B}^*$)

$N_{\mathrm{in}} =$ # inelastic channels — here: 5 ($\Upsilon(1,2,3S)\pi, h_b(1P,2P)\pi$)

Solving the equations

→ The pole terms can be largely decoupled

F.-K. Guo et al., PRD **93** (2016) 074031 [arXiv:1602.00940].

→ No direct (contact type) $Q\bar{Q}\pi \to Q\bar{Q}\pi$ transitions

C. H. et al., PRL **115** (2015) 202001 [arXiv:1507.00382].

$$\beta = \overline{1,\, N_{\mathsf{e}}} \qquad i = \overline{1, N_{\mathsf{in}}}$$

$$\hat{V} = \begin{pmatrix} v_{\alpha\beta}(\boldsymbol{p}, \boldsymbol{p}') & v_{\alpha i}(\boldsymbol{p}, \boldsymbol{k}) \\ v_{j\beta}(\boldsymbol{k}', \boldsymbol{p}') & 0 \end{pmatrix} \qquad \begin{matrix} \alpha = \overline{1\, N_{\mathsf{e}}} \\ j = \overline{1, N_{\mathsf{in}}}, \end{matrix}$$

→ Absorb inelastic channels in effective potential

$$V^{\text{eff.}}_{\alpha\beta} =$$

→ Only $N_{\mathrm{e}} \times N_{\mathrm{e}}$ dimensional problem to be solved (here: 2×2)

→ Only re-scattering terms in production (=pole dominance)

Fitting the data

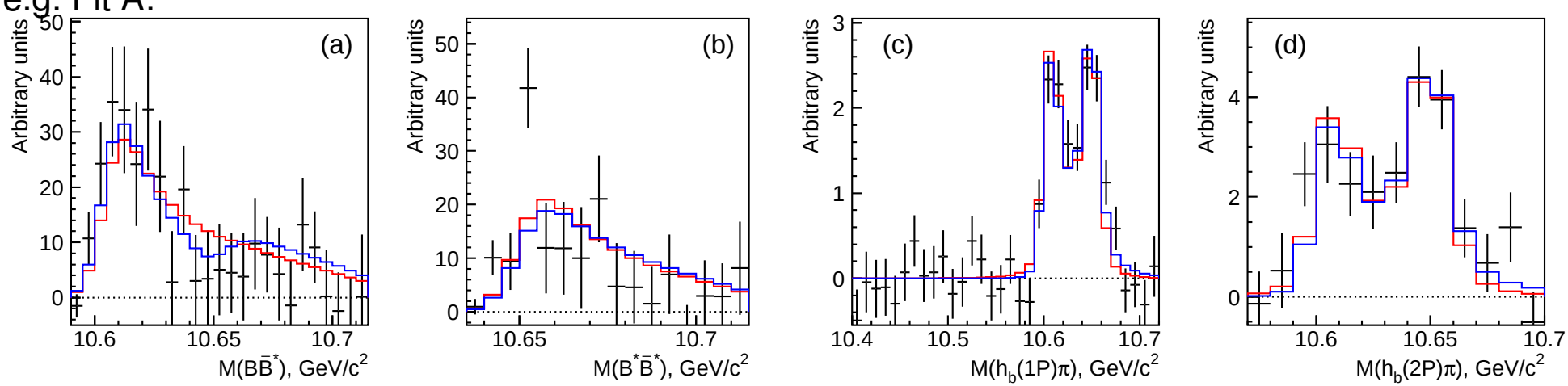
$$\xi = \frac{g_{[\pi\Upsilon(5S)][B^*\bar{B}^*]}}{g_{[\pi\Upsilon(5S)][B\bar{B}^*]}}; R_1 = \frac{g_{[\pi h_b(1P)][B^*\bar{B}^*]}}{g_{[\pi h_b(1P)][B\bar{B}^*]}}; R_2 = \frac{g_{[\pi h_b(2P)][B^*\bar{B}^*]}}{g_{[\pi h_b(2P)][B\bar{B}^*]}}$$

Fit	γ_s [MeV]	γ_t [MeV]	ξ	R_1	R_2	C.L.
A	-86 ± 16	-93 ± 24	-1^*	1^*	1^*	47%
B	35 ± 23	-228 ± 31	-0.83 ± 0.05	1.73 ± 0.21	1.72 ± 0.27	55%

*: Heavy Quark Spin Symmetry imposed

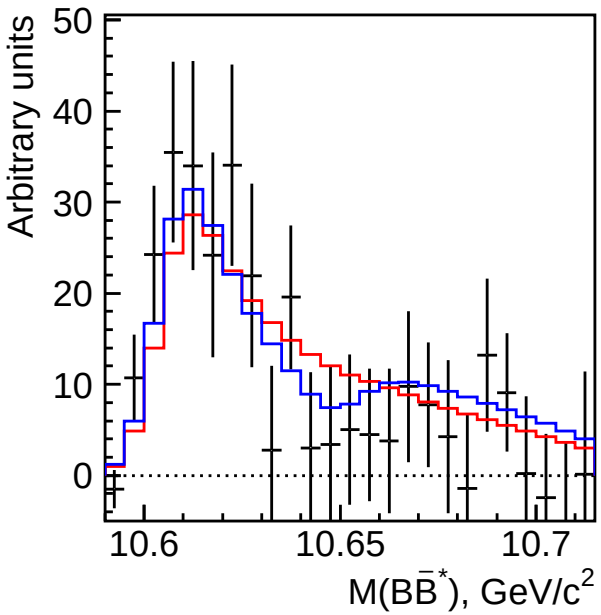
Both fits are acceptable!

e.g. Fit A:



Data: A. Garmash *et al.* [Belle Collaboration], `arXiv:1512.07419` and previous talk.

Discussion



For the original potential:

$$V_{12} \propto \left(\gamma_s^{-1} - \gamma_t^{-1}\right); \; V_{11} = V_{22} \propto \left(\gamma_s^{-1} + \gamma_t^{-1}\right)$$

For fit A: $V_{12} \ll V_{11} = V_{22}$

For fit B: $V_{12} \simeq V_{11} = V_{22}$

V_{12} provides $B^* \bar{B} - B^* \bar{B}^*$ transitions

HQSS imposed$\rightarrow B^* \bar{B}$ and $B^* \bar{B}^*$ decouple

$\longrightarrow$ conservation of light quark spin?

M.B. Voloshin, PRD **93** (2016)074011 [arXiv:1601.02540]

In both fits Z_b and Z_b' (quasi)-virtual states
(poles on unphysical sheet below $\bar{B}^* B$ and $\bar{B}^* B^*$ branch point)

$\rightarrow$must be two–hadron states

Summary and Outlook

→ Combined fits of all relevant channels necessary

→ When proper formalism is applied to $Z_b(10610)$ and $Z_b(10650)$

- ▷ Fit to data works well
- ▷ HQSS constraints can be imposed
- ▷ we find from fit A (with HQSS constraints)

$$\varepsilon_B(Z_b) = (1.51^{+0.76}_{-0.61} \pm i0.11^{+0.03}_{-0.03})\ \text{MeV}, \quad \varepsilon_B(Z_b') = (1.50^{+1.76}_{-0.60} \pm i0.15^{+0.04}_{-0.04})\ \text{MeV},$$

and from fit B (without HQSS constraints)

$$\varepsilon_B(Z_b) = (0.85^{+0.80}_{-0.54} \pm i0.04^{+0.02}_{-0.02})\ \text{MeV}, \quad \varepsilon_B(Z_b') = (1.27^{+0.91}_{-0.69} \pm i1.20^{+0.51}_{-0.43})\ \text{MeV},$$

all on the unphysical sheets→two–hadron states

Result converted from BW-fit R. Mizuk, private communication

$$\varepsilon_B(Z_b) = (-2.6 \pm 2.1 - i(9.2 \pm 1.2))\ \text{MeV}, \quad \varepsilon_B(Z_b') = (-1.8 \pm 1.7 - i(5.8 \pm 1.1))\ \text{MeV},$$

Do not sum Breit–Wigners!