Quarkonium spectral functions from lattice NRQCD

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Based on
S. Kim, PP, A. Rothkopf, PRD91 (2015) 054511
and
S. Kim, PP, A. Rothkopf, work in progress

Up to which temperatures can different quarkonia states survive ?

Do we see the expected sequential suppression pattern in QCD ?

Quarkonium Workshop 2016 (QWG11), PNNL, June 6-10, 2016

Why NRQCD ?

Quarkonia to a fairly good approximation are non-relativistic bound state

$$p_Q \sim M_Q v \ll M_Q$$

EFT approach: integrate the physics at scale of the heavy quark mass

NRQCD is the EFT at scale << M_Q

Heavy quark fields are non-relativistic Pauli spinors:

$$L_{NRQCD} = \psi^\dagger \left(D_\tau - \frac{D_i^2}{2M_Q} \right) \psi + \chi^\dagger \left(D_\tau + \frac{D_i^2}{2M_Q} \right) \chi + ... + \tfrac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Advantages:

- the large quark is not a problem for lattice calculations, lattice study of bottomonium is feasible (usually $a M_Q << 1$, which is challenging)
- The structure of the spectral function is simpler => more sensitivity to the bound state properties
- Quarkonium correlators are not periodic and can be studied at larger time extent ($=1/T$) => more sensitivity to bound state properties

NRQCD on the Lattice

Inverse lattice spacing provides a natural UV cutoff for NRQCD provided $a^{-1} \leq 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$G(x,\tau+a) = U_4^\dagger(1 - \frac{p^2}{2M_Q}\Delta\tau)G(x,\tau), \;\; \Delta\tau = a/n$$

well behaved if $naM_Q < 3$

Davies, Thacker, PRD 45 (1992) 915

$$D(\tau) = \sum_x \langle O(x,\tau) G_{x\tau} O^\dagger(x_0,\tau_0) G^\dagger_{x\tau}\rangle_{med} \qquad O(^3S_1;x,\tau) = \sigma_i, \quad O(^3P_1;x,\tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

Thacker, Lepage, PRD43 (1991) 196

The energy levels in NRQCD are related to meson masses by a constant lattice spacing dependent shift, e.g.

$$M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\rm shift}(a)$$

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied by varying $a \leftrightarrow \beta = 10/g^2$ Bazavov et al, PRD85 (2012) 054503

$$\Rightarrow 140\text{MeV} \leq T \leq 407\text{MeV}$$

$$2.759 \geq aM_b \geq 0.954 \text{ (ok if } n = 2,\ 4)$$
$$0.757 \geq aM_c \geq 0.427 \text{ (ok if } n \geq 8)$$

Bayesian Reconstruction of spectral functions

$$D(\tau) = D(\mathbf{p}=0,\tau) = \sum_{\mathbf{x}} D(\mathbf{x},\tau) = \int_{-2M_q}^{\infty} d\omega\, e^{-\omega\tau}\, \rho(\omega)$$

Discretize the integral $D_i^\rho = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$ and find ρ_l

using Bayesian approach, i.e. maximizing

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I]$$

Likelihood:

$$P[D|\rho I] = \exp(-L)$$

$$L[\rho] = \frac{1}{2}\sum_{ij}(D_i - D_i^\rho)C_{ij}(D_j - D_j^\rho)$$

Prior probability:

$$P[\rho|I] = \exp[S]$$

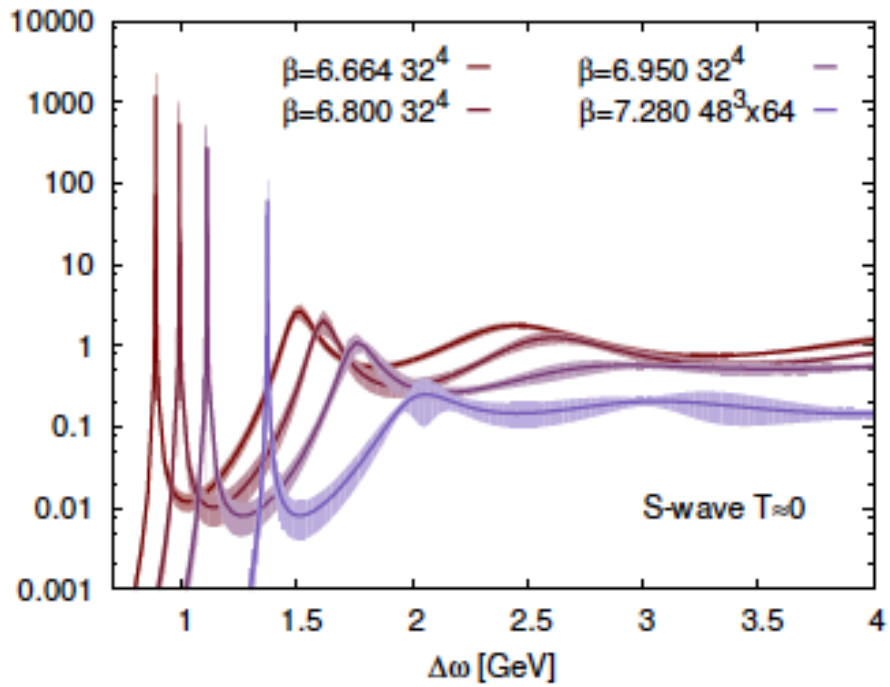
$$S[\rho] = \alpha \sum_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)\Delta\omega_l.$$

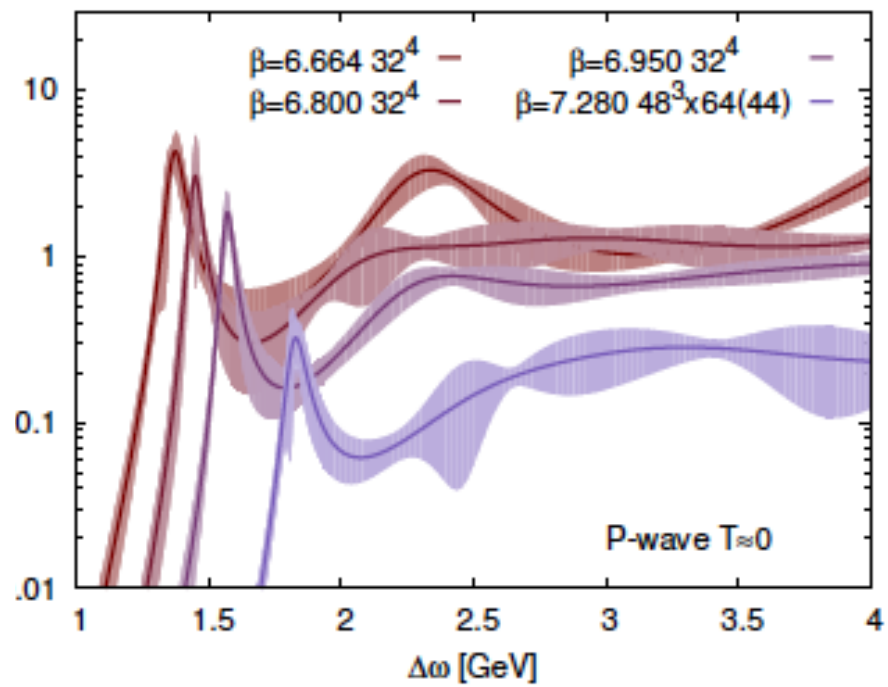
no restriction on the search space
no flat directions

Different from MEM !

Burnier Rothkopf, PRL 111 (2013) 182003

Bottomonium spectral functions at T=0





Well resolved Υ ground state peak

Acceptable resolution for χ_b state

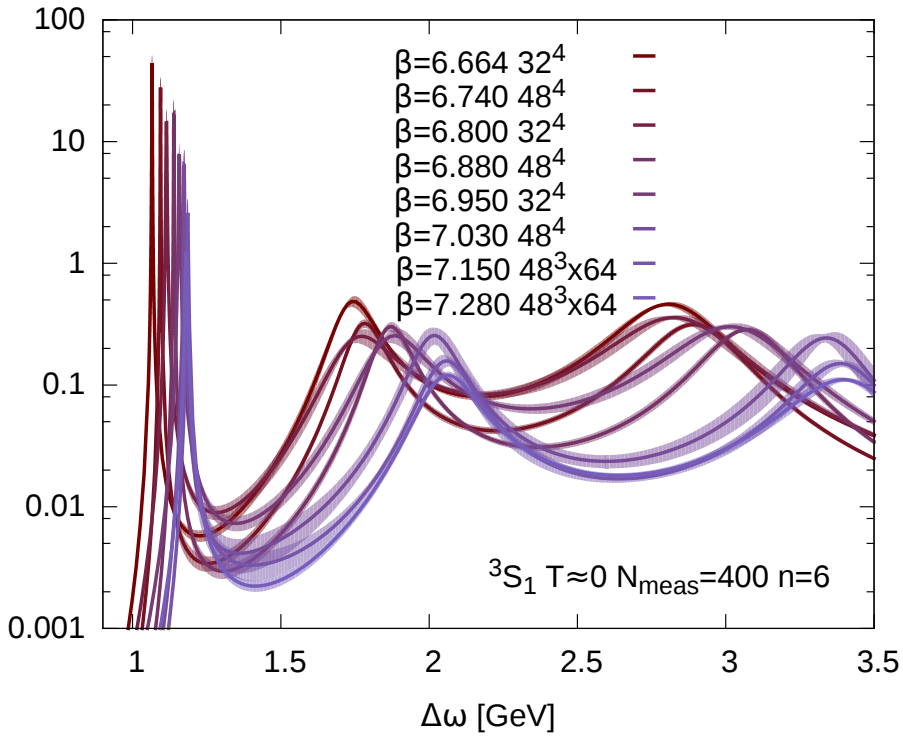
But excited states, Υ', Υ'', χ_b' cannot be resolved well

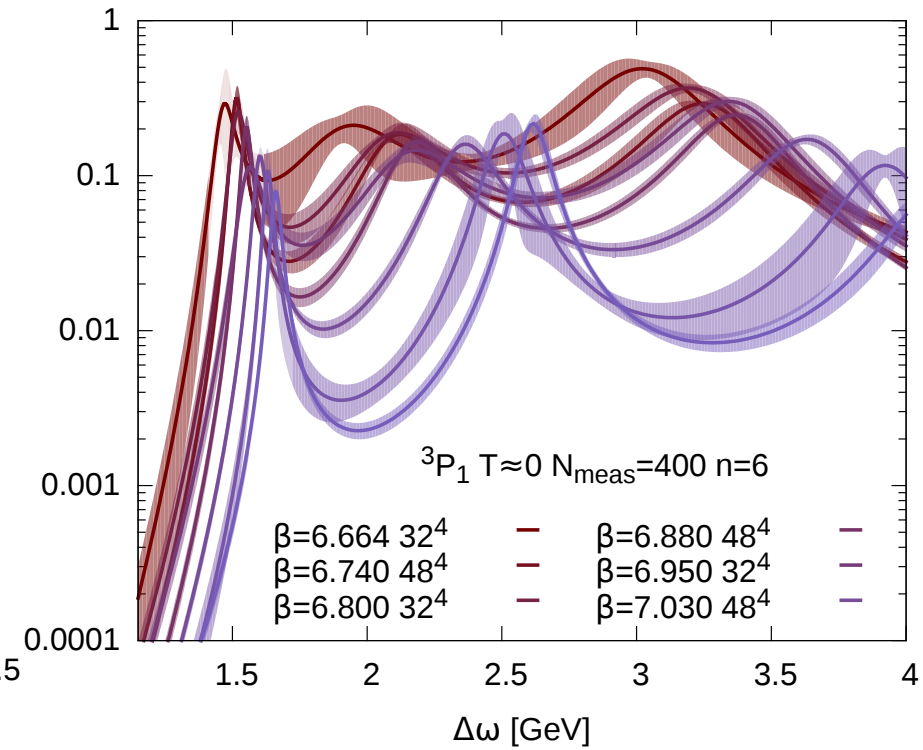
Define the NRQCD energy shift $C_{\rm shift}(a)$ by fixing the Υ peak to PDG

$$E_\Upsilon + C_{\rm shift}(a) = 9.46030 \text{ GeV}$$

$\Rightarrow$ prediction for mass of other states: η_b, χ_{b0}, χ_{b1}, h_b

Charmonium spectral functions at T=0





Well resolved J/ψ peak

Only the position of the χ_{c1} state can be reliably determined

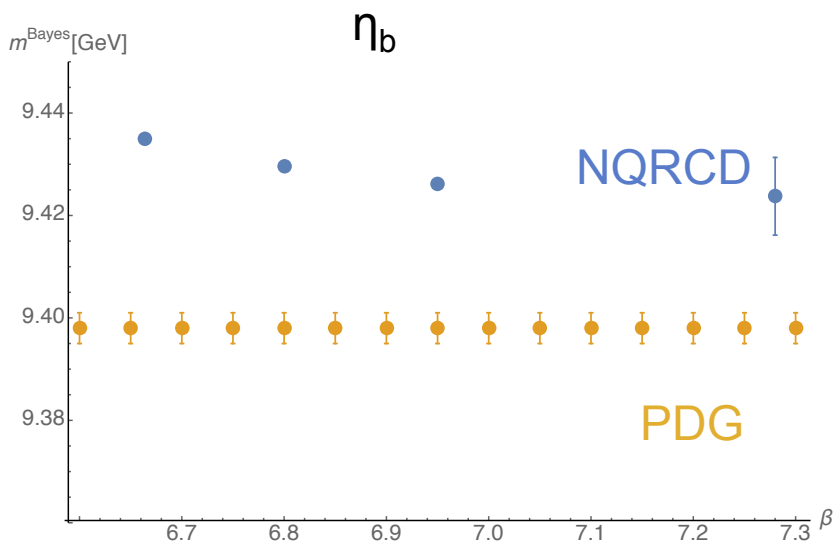
Excited states cannot be determined, artifacts at $\Delta\omega > 3$ GeV

Define the NRQCD energy shift $C_{\rm shift}(a)$ by fixing the J/ψ peak to PDG

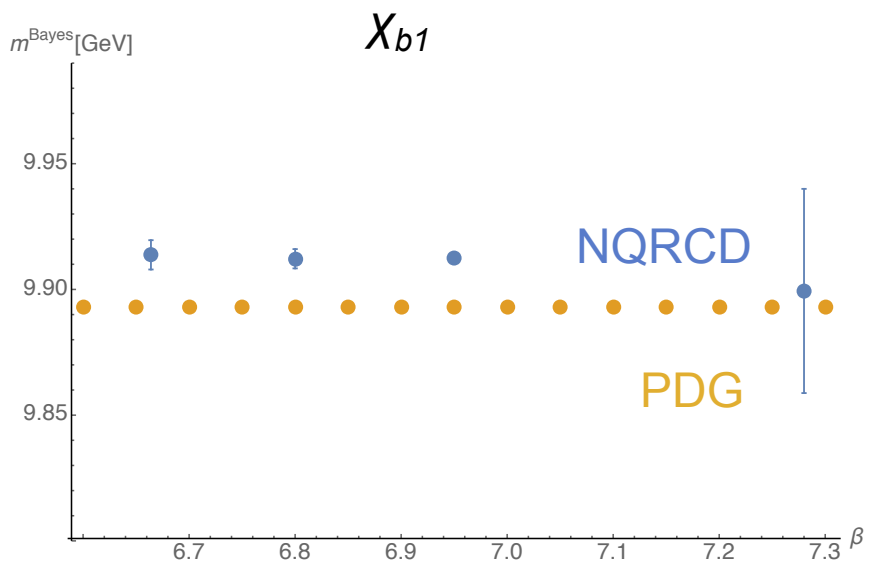
$$E_{J/\psi} + C_{\rm shift}(a) = 3.097 \text{ GeV}$$

$\Rightarrow$ prediction for mass of other states: η_c, χ_{c0}, χ_{c1}, h_c

How Well NQRCD Works for Bottomonium ?

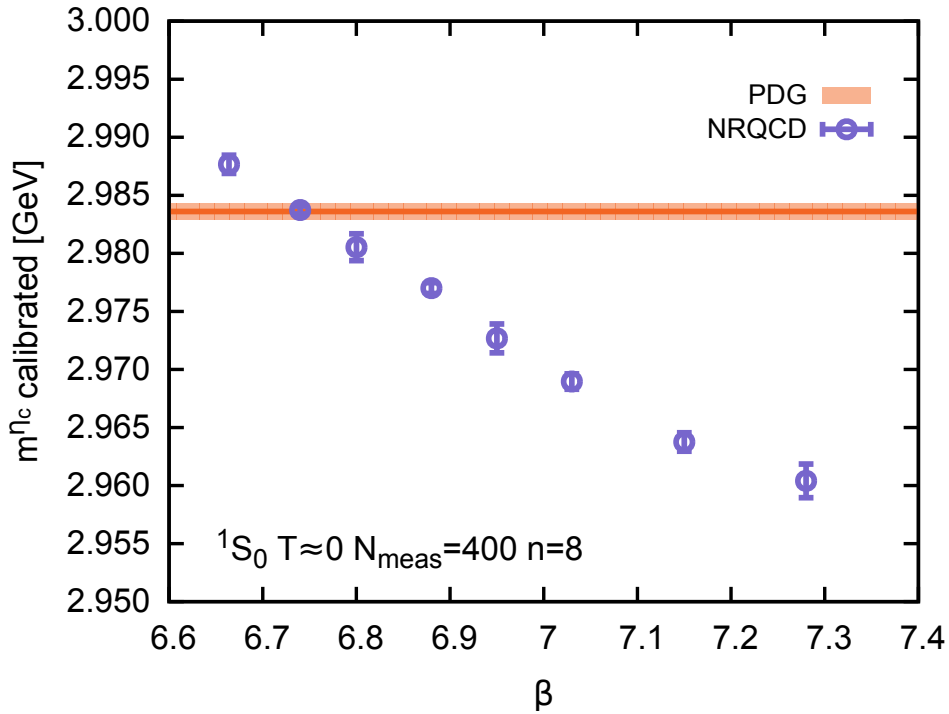


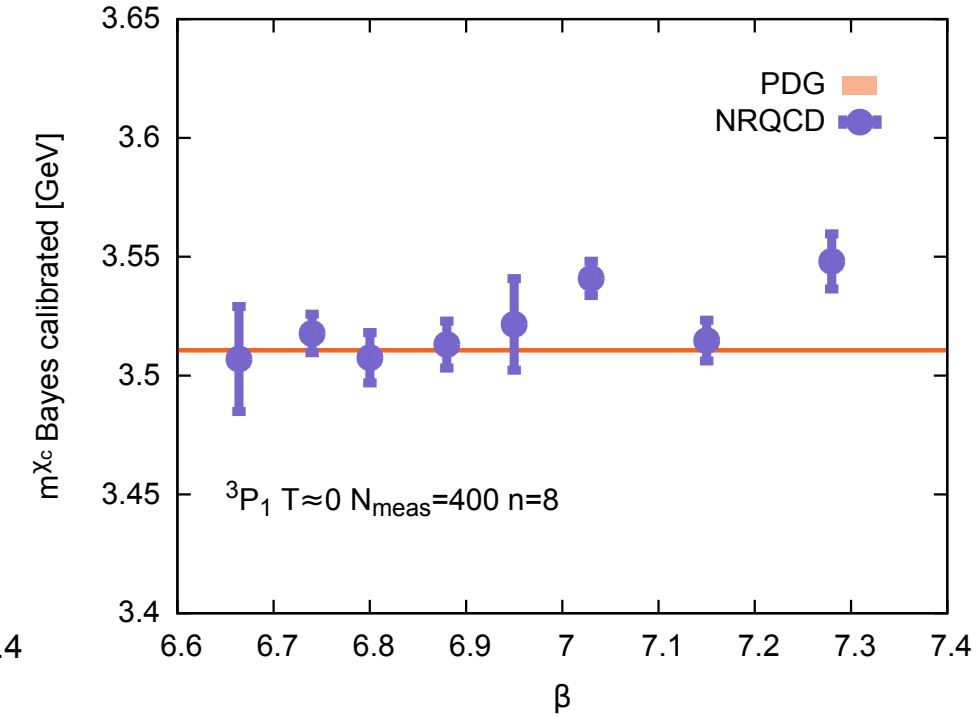
NRQCD can reproduce the hyperfine splitting in bottomonium with accuracy < *20-40* MeV depending on the lattice spacing



NRQCD can reproduce the *1P-1S* splitting in bottomonium with accuracy < *15* MeV

How Well NQRCD Works for Charmonium ?

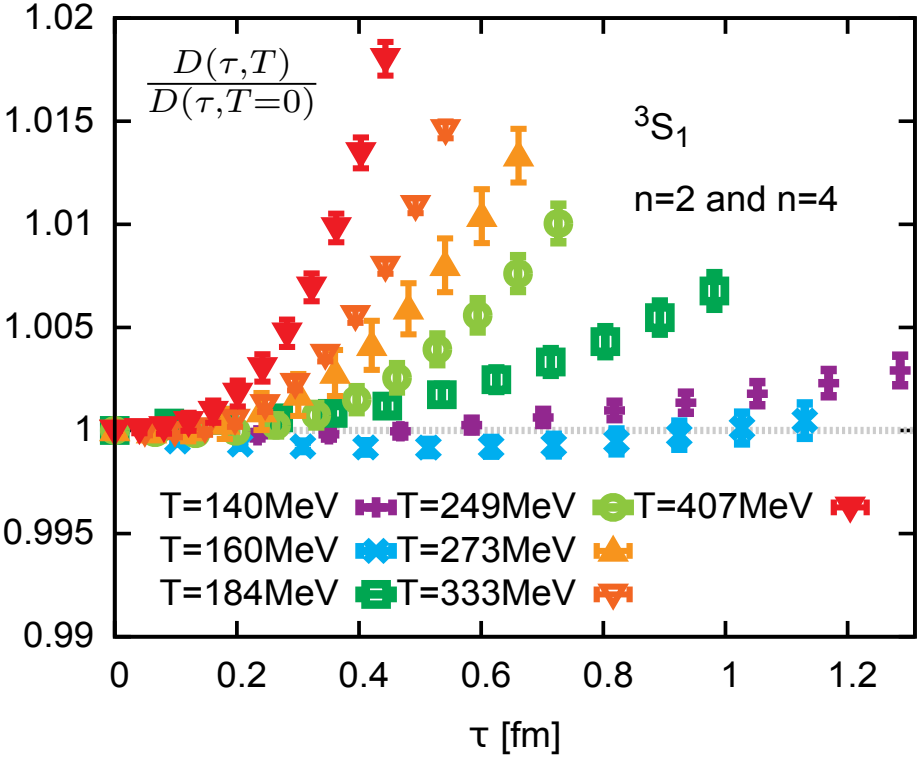


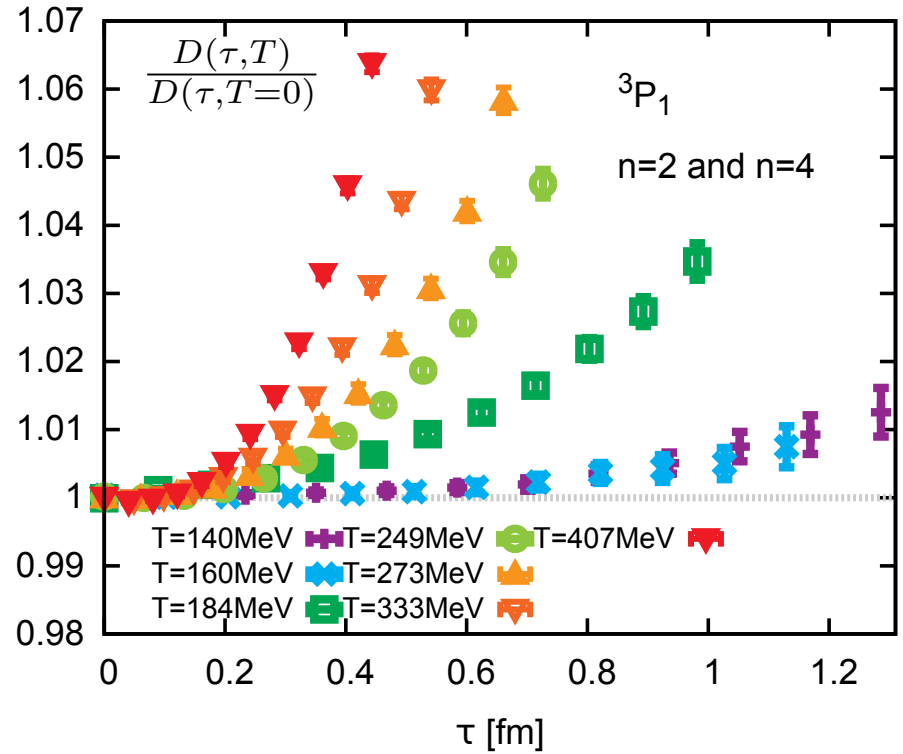


NRQCD can reproduce the hyperfine splitting in charmonium with an accuracy < *40* MeV

NRQCD can reproduce the *1P-1S* splitting in charmonium well for lattice spacing a > 0.08fm

Temperature dependence of the bottomonium correlators



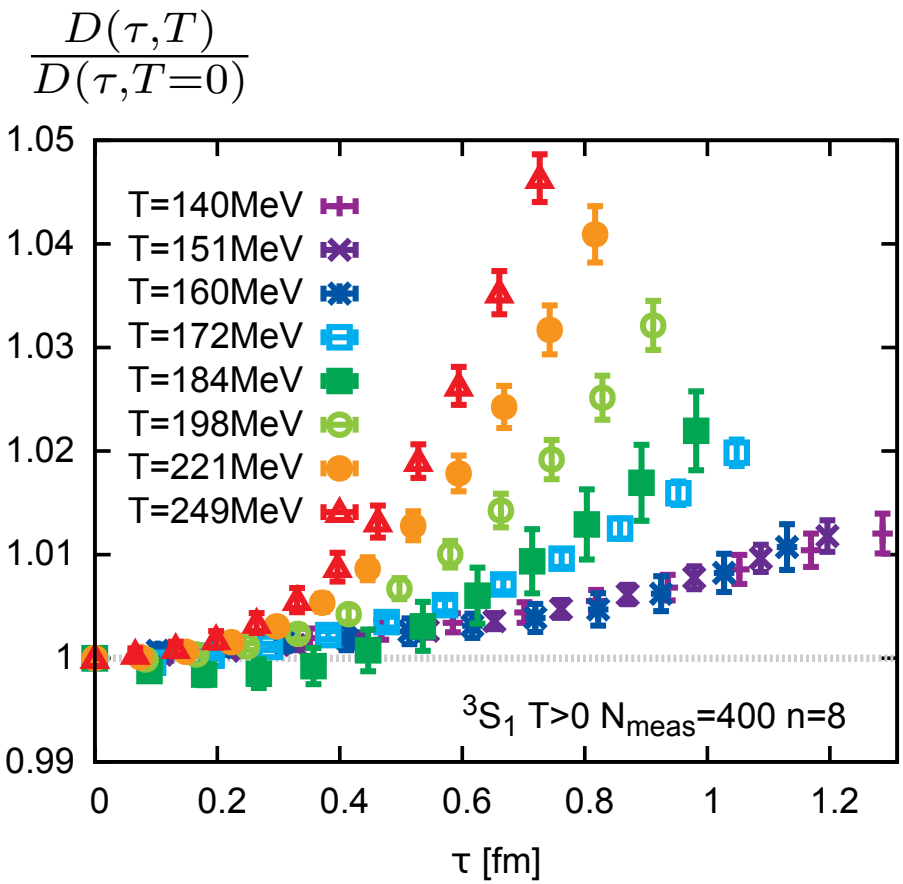


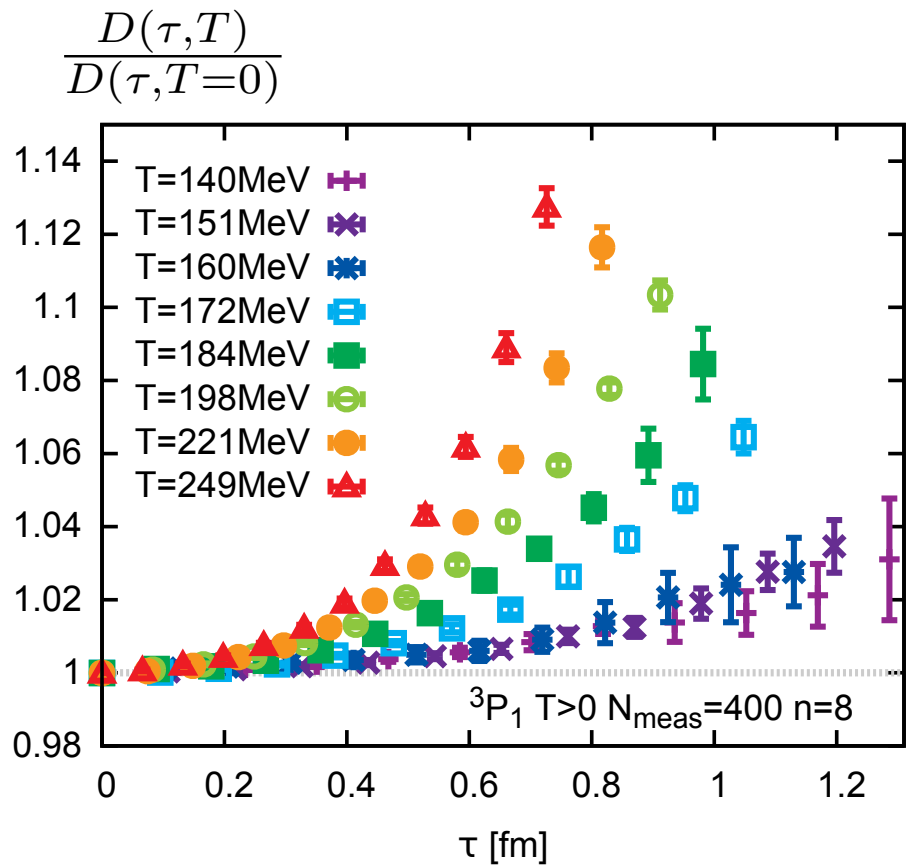
change in Υ correlator $< 2\%$

change in χ_{b1} correlator $< 7\%$

$\Rightarrow$ hints for sequential melting pattern: stronger medium modification of χ_{b1} spectral function than for Υ spectral function

Temperature dependence of the charmonium correlators





change in J/ψ correlator $< 5\%$

change in χ_{c1} correlator $< 12\%$

$\Rightarrow$ hints for sequential melting pattern:
changes in the J/ψ correlator are about the same as in the χ_b correlator (same size); changes in the χ_c correlators are factor of two larger

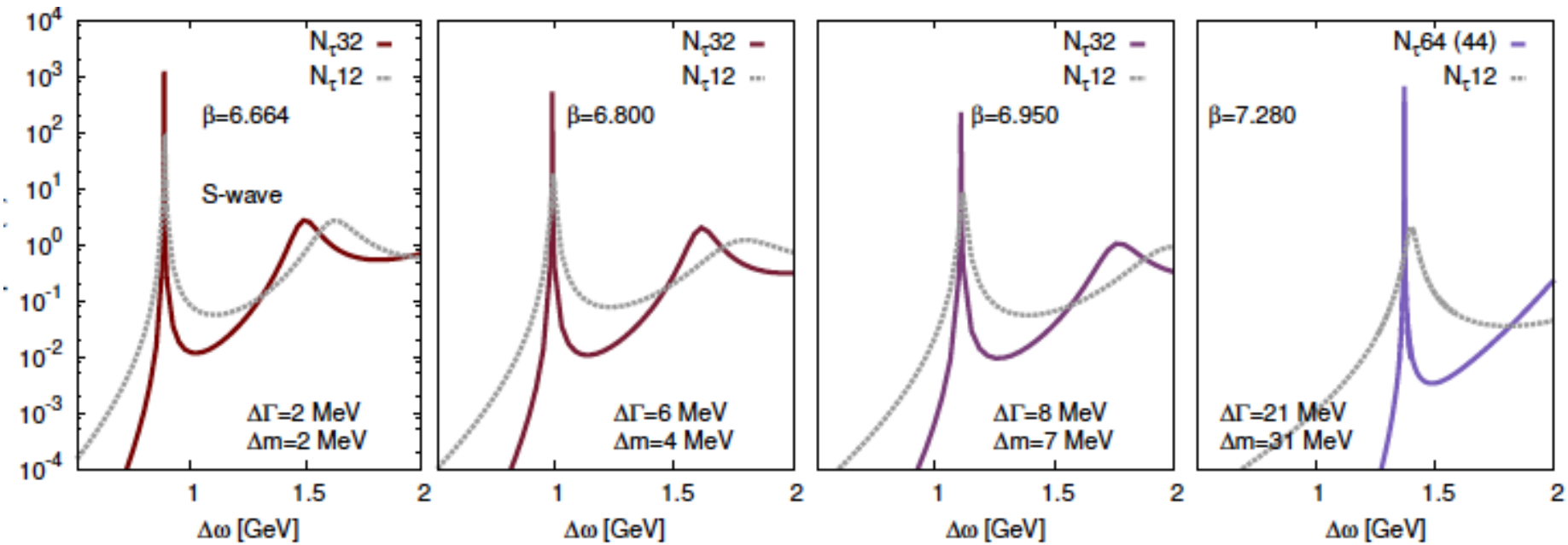
Reconstructing Spectral Functions at T>0

Two main problems:

1) $\tau < 1/T \Rightarrow$ limited temporal extent at high T

2) relatively small number of time slices ($N_\tau = 12$ in our study)

Study these effects at $T = 0$ by using only the first 12 data points:



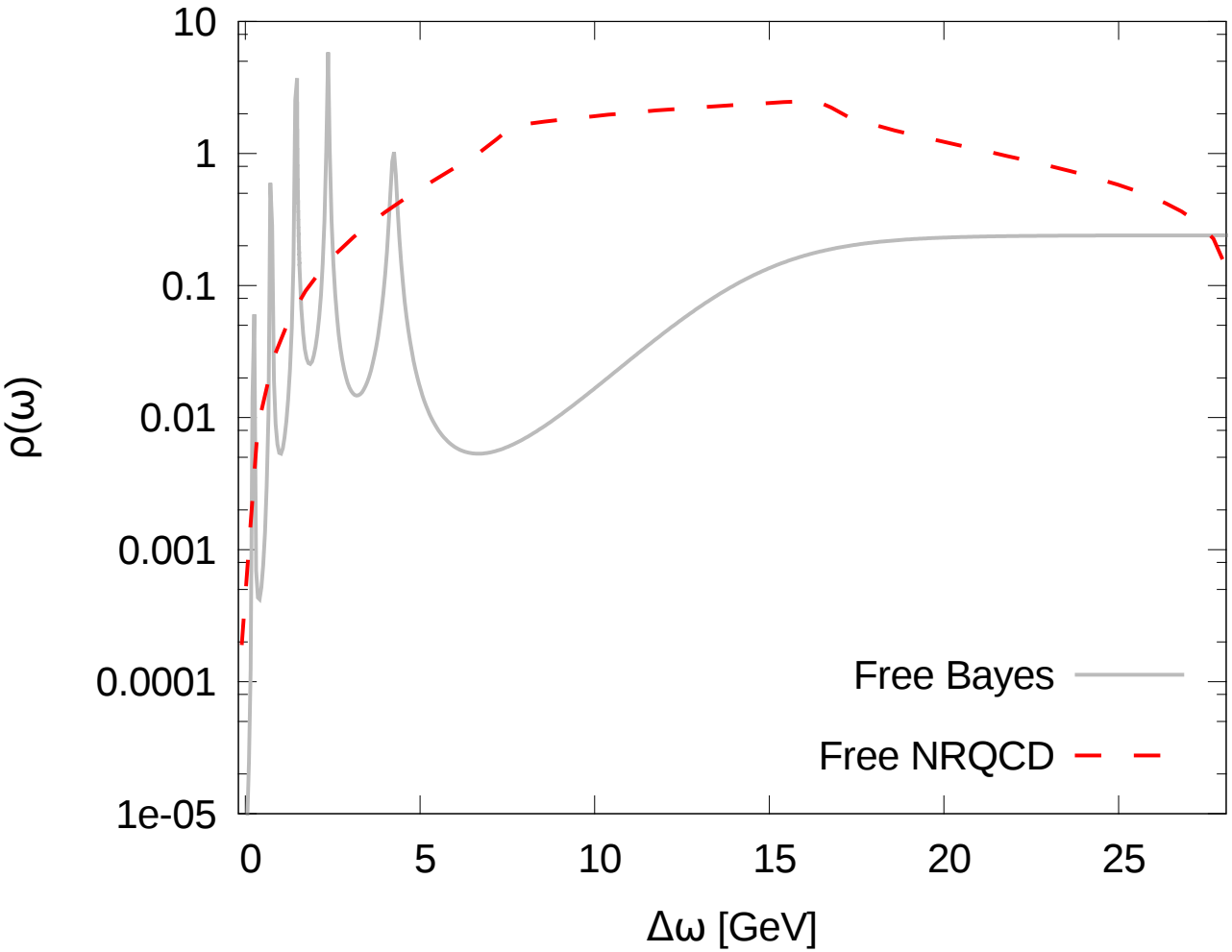
Decreasing $\tau_{max} = 1/T$ leads to broadening of the bound state peak

(to be taken into account in comparison $T = 0$ and $T > 0$ spectral functions)

Reconstructing Spectral Functions at T>0 II

Because of the limited number of the data points the reconstructed spectral function may not be smooth

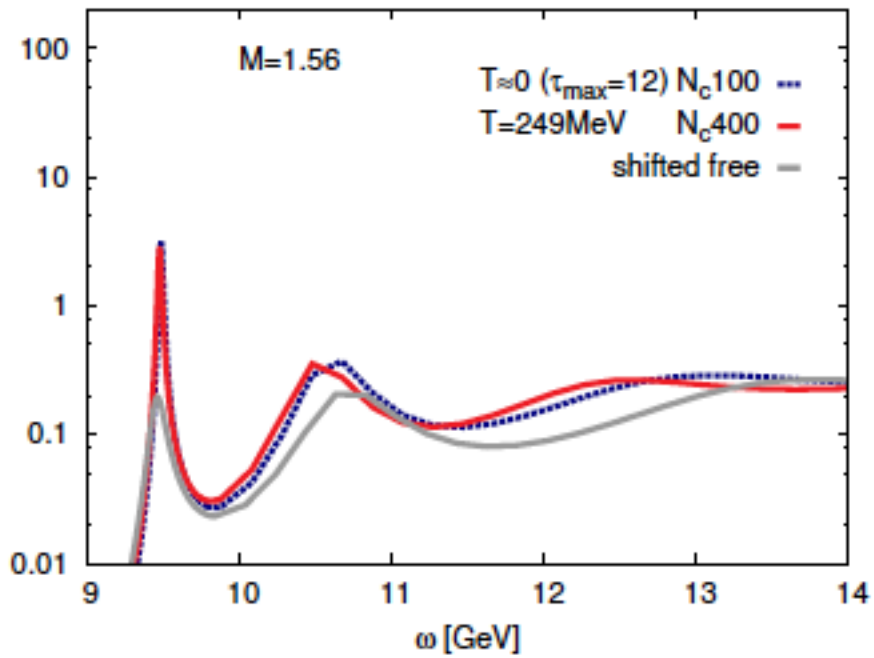
Consider free correlators (smooth spectral functions):

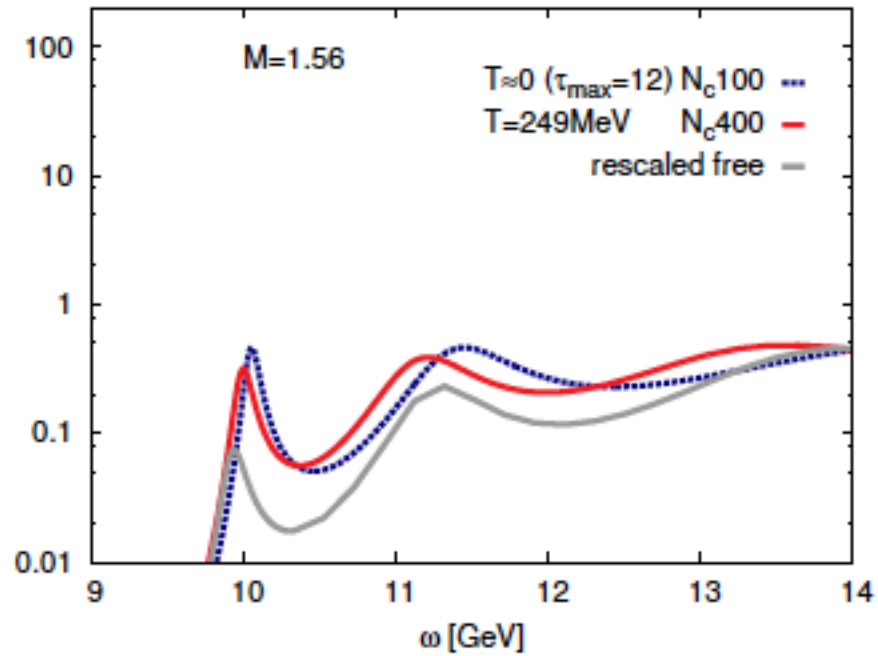


oscillation (ringing) around the free result

Bottomonium Spectral Functions at T>0

Compare $T = 0$, $T > 0$ and free spectral functions reconstructed using the same systematics ($\tau_{max} = 1/T$ and $N_{data} = 12$)





Both Υ and χ_b survive up to temperature $T > 249$ MeV

Summary

- Both charmonium and bottomonium properties at $T > 0$ can be studied reliably using lattice NRQCD

- Systematic effects in the reconstructed spectral functions at $T > 0$ should be carefully studied through comparison to the $T = 0$ spectral functions and free spectral functions obtained under the same conditions

- Combined analysis of the T-dependence of the correlation functions and study of the spectral functions suggests a sequential dissociation (melting) pattern

$$T_d(\chi_c) < T_d(J/\psi) \simeq T_d(\chi_b) \ll T_d(\Upsilon)$$

in agreement with potential model calculations

PP, Miao, Mocsy, NPA855 (2011) 125

and the study of spatial charmonium correlators

Bazavov et al, PRD91 (2015) 054503