The Polyakov loop at order g^5

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Bibliography

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Polyakov loop in 2+1 flavor QCD from low to high temperatures
Phys. Rev. D93 (2016) 114502 `arXiv:1603.06637`

(2) M. Berwein, N. Brambilla, P. Petreczky and A. Vairo
Polyakov loop at next-to-next-to-leading order
Phys. Rev. D93 (2016) 034010 `arXiv:1512.08443`

(3) M. Berwein, N. Brambilla and A. Vairo
Renormalization of Loop Functions in QCD
Phys. Part. Nucl. 45 (2014) 656 `arXiv:1312.6651`

(4) N. Brambilla, J. Ghiglieri, P. Petreczky and A. Vairo
The Polyakov loop and correlator of Polyakov loops at next-to-next-to-leading order
Phys. Rev. D82 (2010) 074019 `arXiv:1007.5172`

Polyakov loop

Polyakov loop and free energy

Polyakov loop, L, and free energy of a static quark, F_Q, in a general representation R of SU(N) of dimension d_R:

$$L = \frac{1}{d_R}\,\mathrm{Tr}\left\langle \mathcal{P}\exp\left[\mathrm{ig}\int_0^{1/\mathrm{T}} \mathrm{d}\tau\,\mathrm{A}_0(\tau,\mathrm{x})\right]\right\rangle \equiv \mathrm{e}^{-\mathrm{F_Q}/\mathrm{T}}$$

- For $n_f = 0$ is an order parameter for deconfinement.
- For $n_f > 0$ at high temperatures it is a measure of the screening properties of the deconfined medium.
- It enters in all quark-antiquark correlators and free energies: Polykov loop correlators, Wilson loops, ...

The Polyakov loop at $\mathcal{O}(g^3)$

$$L = 1 + \frac{C_R \alpha_s m_D}{2T}, \qquad F_Q = -\frac{C_R \alpha_s m_D}{2}$$

where C_R is the quadratic Casimir of the representation R.
The Debye mass m_D is given by

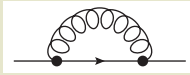
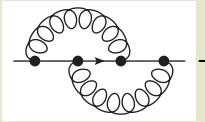
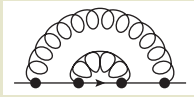
$$\Pi_{00}(|\mathbf{k}| \ll T) \approx m_D^2 = \frac{C_A + T_F n_f}{3} g^2 T^2$$

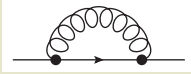
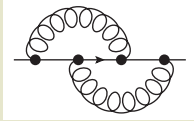
In the weak coupling one assumes the hierarchy of scales

$$T \gg m_D \sim gT \gg m_M \sim g^2 T$$

where m_M is the magnetic mass.

Exponentiation

$L = 1 + C_R$  $+ C_R^2$ $+ C_R\left(C_R - \frac{C_A}{2}\right)$  $+ C_R^2$  $+ \dots$

$= \exp\left[C_R\right.$  $\left. - \frac{1}{2} C_R C_A\right.$  $\left. + \dots\right] = \exp\left(D_1 + D_2 + \dots\right)$

- Dots stand for order g^6 contributions.
- The free energy is proportional to C_R: Casimir scaling.

$$D_1 = -\frac{C_R g^2}{2T} \int_k D_{00}(0, \boldsymbol{k})$$

$$D_2 = -\frac{C_R C_A g^4}{4T} \int_{k,\, q} \left[\frac{1}{12T} D_{00}(0, \boldsymbol{k}) D_{00}(0, \boldsymbol{q}) - \sum_{k_0}{}' \frac{1}{k_0^2} D_{00}(K)\big(2D_{00}(0, \boldsymbol{q}) - D_{00}(k_0, \boldsymbol{q})\big)\right]$$

The Polyakov loop at $\mathcal{O}(g^4)$

$$L = 1 + \frac{C_R \alpha_s}{2} \frac{m_D}{T} + \frac{C_R \alpha_s^2}{2} \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2} \right) - n_f \ln 2 \right]$$

- The logarithm, $\ln m_D^2/T^2$, signals that an infrared divergence at the scale T has canceled against an ultraviolet divergence at the scale m_D.

- Burnier Laine Vepsäläinen JHEP 1001 (2010) 054
  Brambilla Ghiglieri Petreczky Vairo PR D82 (2010) 074019

Comparison with the literature

In 1981, Gava and Jengo obtained at $\mathcal{O}(g^4)$ (for $n_f = 0$):

$$L_{\mathrm{GJ}} = 1 + \frac{C_R \alpha_{\mathrm{s}}}{2} \frac{m_D}{T} + \frac{C_R C_A \alpha_{\mathrm{s}}^2}{2} \left(\ln \frac{m_D^2}{T^2} - 2 \ln 2 + \frac{3}{2} \right)$$

The result is incorrect. The origin of the error can be traced back to not having resummed the Debye mass in the temporal gluons contributing to the static gluon self energy.

- `Gava Jengo PLB 105 (1981) 285`

Our result agrees with the determination of Burnier, Laine and Vepsäläinen, who use a dimensionally reduced EFT framework in a covariant or Coulomb gauge.

- `Burnier Laine Vepsäläinen JHEP 1001 (2010) 054`

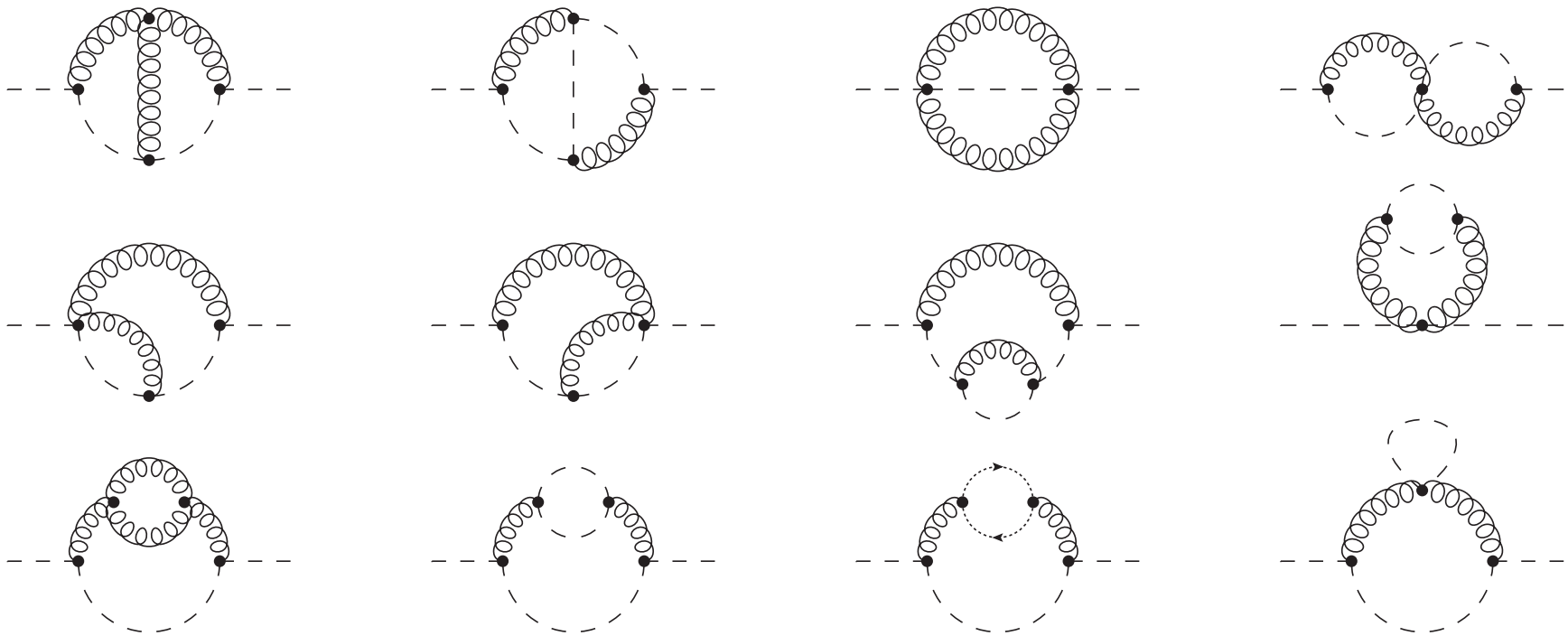
Contributions from the scales T and m_D at $\mathcal{O}(g^5)$

After charge renormalization:

$$(D_1 + D_2)\Big|_{g^5,\,T} = \frac{3C_R\alpha_s^2 m_D}{16\pi T}\left[3C_A + \frac{4}{3}T_F n_f\,(1 - 4\ln 2) + 2\beta_0\left(\gamma_E + \ln\frac{\mu}{4\pi T}\right)\right] - \frac{C_R C_F T_F n_f \alpha_s^3 T}{2m_D}$$

$$D_1\Big|_{g^5,\,m_D} = -\frac{C_R C_A^2 \alpha_s^3 T}{m_D}\left[\frac{89}{48} + \frac{\pi^2}{12} - \frac{11}{12}\ln 2\right]$$

$\Pi^{(2)}_{m_D}(0, k \sim m_D)$



The Polyakov loop at $\mathcal{O}(g^5)$

$$
\begin{aligned}
\ln L = {} & \frac{C_R \alpha_{\rm s}(\mu)\, m_D}{2T} + \frac{C_R \alpha_{\rm s}^2}{2}\left[C_A\left(\frac{1}{2} + \ln\frac{m_D^2}{T^2}\right) - 2T_F n_f \ln 2\right] \\
& + \frac{3 C_R \alpha_{\rm s}^2 m_D}{16\pi T}\left[3C_A + \frac{4}{3}T_F n_f\,(1 - 4\ln 2) + 2\beta_0\left(\gamma_E + \ln\frac{\mu}{4\pi T}\right)\right] \\
& - \frac{C_R C_F T_F n_f \alpha_{\rm s}^3 T}{2 m_D} \\
& - \frac{C_R C_A^2 \alpha_{\rm s}^3 T}{m_D}\left[\frac{89}{48} + \frac{\pi^2}{12} - \frac{11}{12}\ln 2\right]
\end{aligned}
$$

- Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

Checks

- Feynman gauge and Coulomb gauge.
- Static gauge: $\partial_0 A_0 = 0$:

$$L = \exp\left(\frac{igA^0(\mathbf{x})}{T}\right) =$$

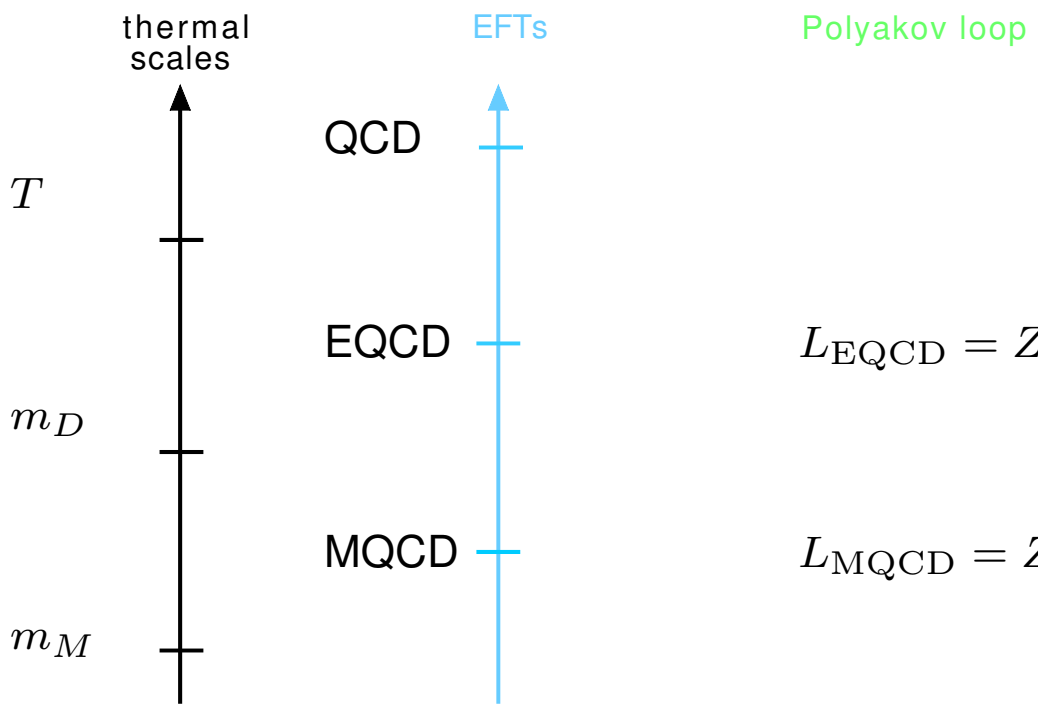
- Phase-space Coulomb gauge:

$$e^{-S} = \exp\left[-\int_0^{1/T} d\tau \int d^3x \left(\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{2}F^a_{0i}F^a_{0i}\right)\right]$$
$$= \mathcal{N}^{-1}\int \mathcal{D}E_i \exp\left[-\int_0^{1/T} d\tau \int d^3x \left(\frac{1}{4}F^a_{ij}F^a_{ij} + iE^a_i F^a_{0i} + \frac{1}{2}E^a_i E^a_i\right)\right]$$

- Andrasi EPJ C37 (2004) 307

- Dimensional reduced effective field theories.

Dimensional reduced EFTs



$$L_{\mathrm{EQCD}} = Z_0^E - Z_2^E \frac{g^2}{2d_R T} \mathrm{Tr}\, \langle \tilde{\mathrm{A}}_0^2 \rangle + \ldots$$

$$L_{\mathrm{MQCD}} = Z_0^M + \frac{Z_1^M}{2m_D^3} \langle \tilde{F}_{ij}^a \tilde{F}_{ij}^a \rangle + \ldots$$

- The Polykov loop may be calculated relying mostly on known results.
  - `Braaten Nieto PR D53 (1996) 3421`
    `Kajantie Laine Rummukainen Shaposhnikov NP B503 (1997) 357, ...`
- Non-perturbative contributions carried by m_M are of order g^7.

Magnetic mass contributions

MQCD shows that magnetic mass contributions appear at $\mathcal{O}(g^7)$.

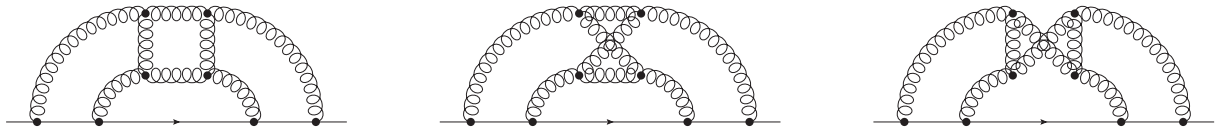
- At order g^5 the following two diagrams cancel when the spatial gluon carries a momentum of order m_M:

- The explicit cancellation of the magnetic mass contributions at order g^6 has also been checked.

Casimir scaling

Casimir scaling holds up to $\mathcal{O}(g^7)$ (including m_M contributions).

Possible Casimir scaling violations may happen at $\mathcal{O}(g^8)$, through diagrams like

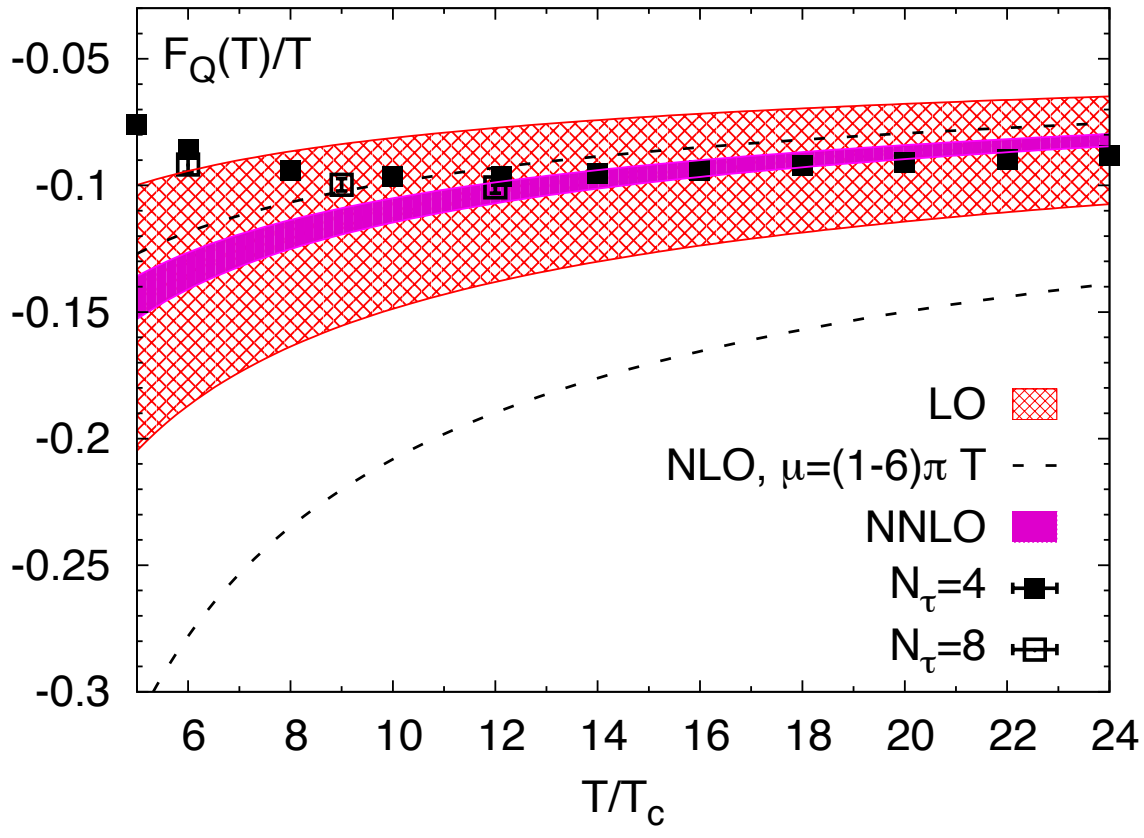


+ 4 gluon vertex diagrams + light quark loop diagrams.

- Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

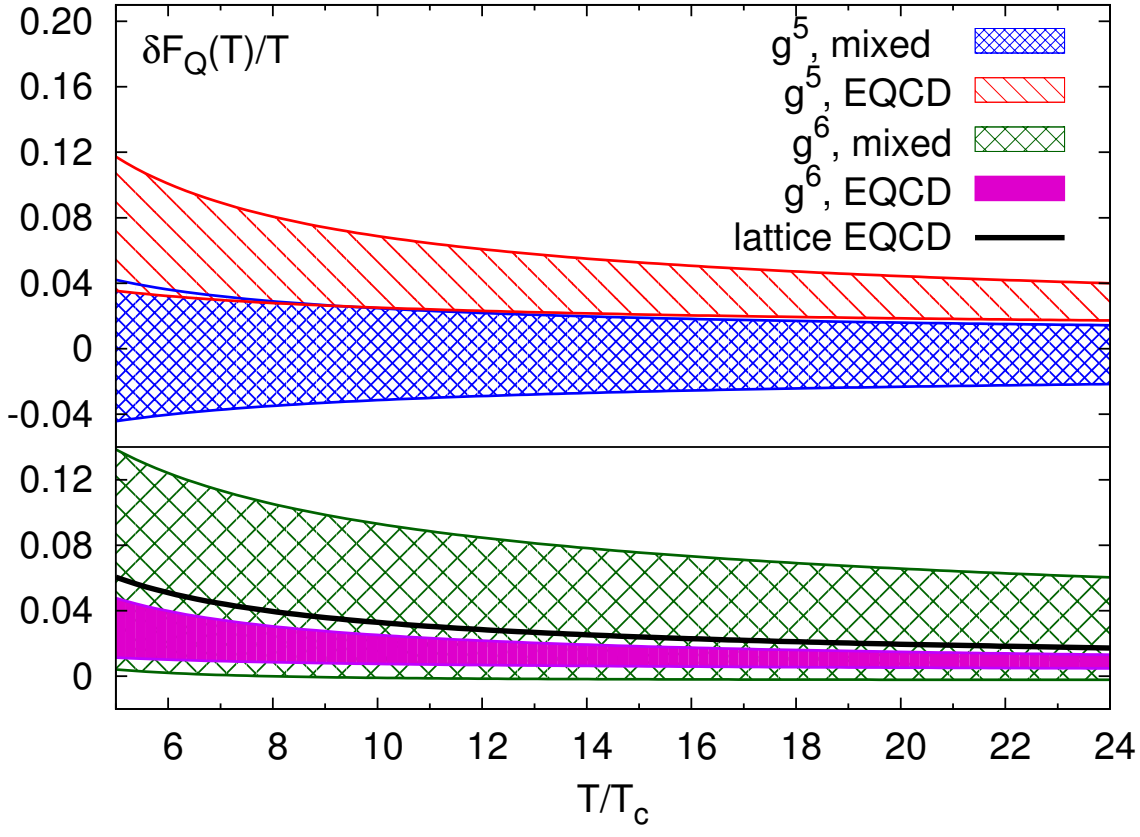
Comparison with lattice

Free energy vs quenched lattice data



- Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

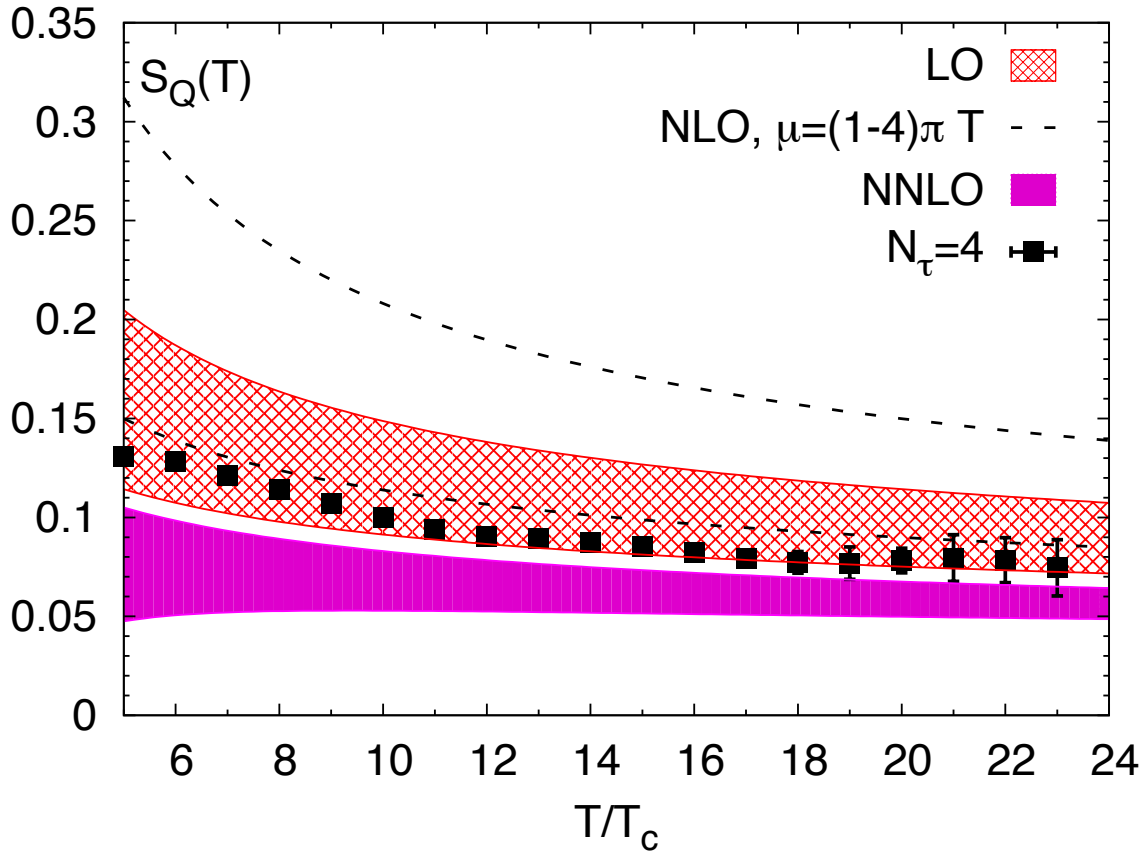
$\delta F_Q/T$ at $\mathcal{O}(g^5)$ and $\mathcal{O}(g^6)$



- Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

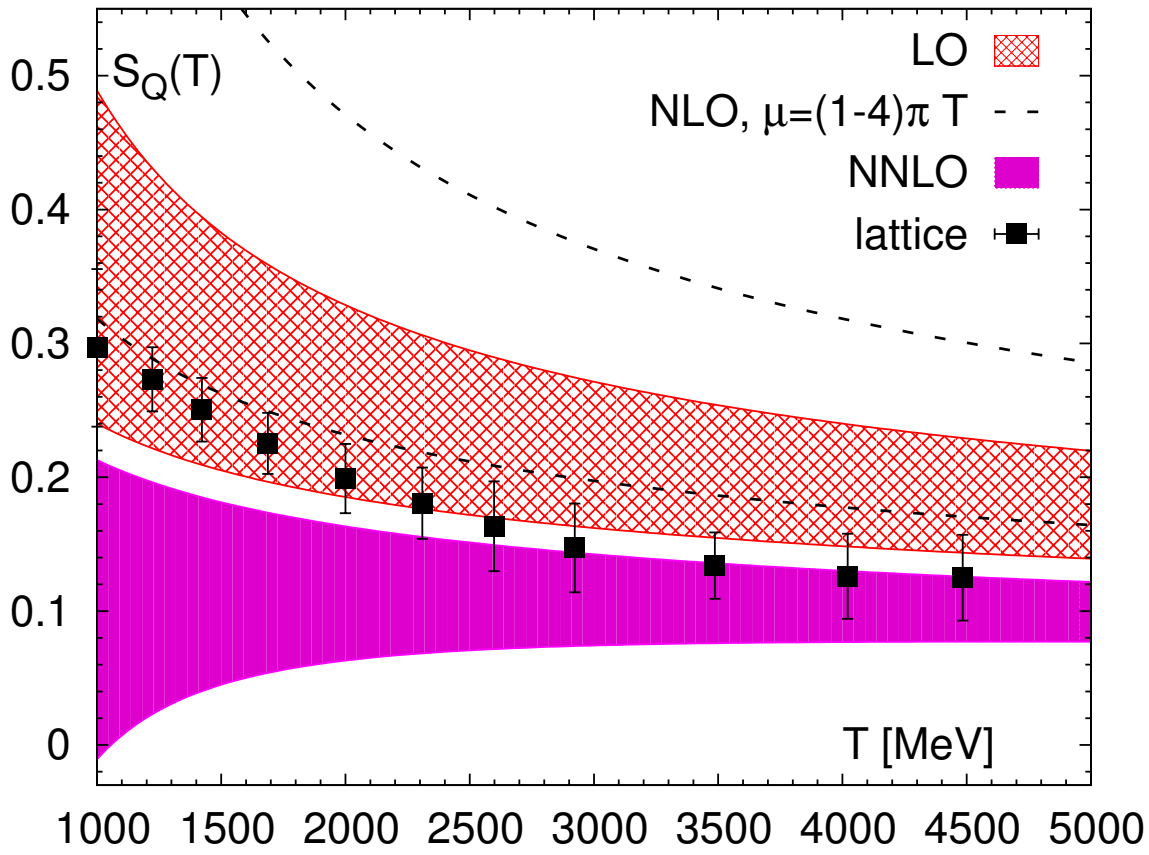
Entropy vs quenched lattice data

The entropy does not depend on the normalization shift: $S_Q = -\frac{\partial F_Q(T)}{\partial T}$.



- Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

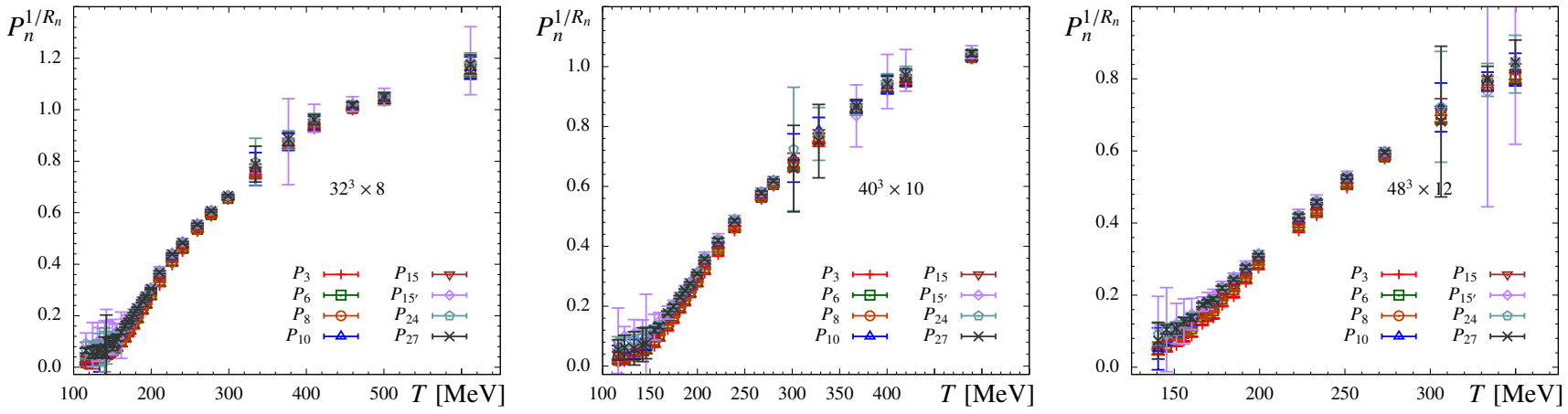
Entropy vs 2+1 flavor lattice data



Position of the entropy peak: $T_S = 153^{6.5}_{-5}$ MeV.

- Bazavov Brambilla Ding Petreczky Schadler Vairo Weber PR D93 (2016) 114502

Casimir scaling



- Petreczky Schadler PR D92 (2015) 094517

Outlook

- The computation of the $\mathcal{O}(g^6)$ is feasible in perturbation theory. It is the last missing piece of the perturbative expansion of the Polyakov loop. Non-perturbative contributions from the magnetic mass are of $\mathcal{O}(g^7)$. It may conclusively bring the weak-coupling expansion in agreement with the lattice data at high temperatures.
- It would be important to have a definitive weak-coupling estimate of the Casimir scaling violation. This may happen at $\mathcal{O}(g^8)$, but we do not have a proof that these contributions do not vanish.